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GATE
Electrical Engineering

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MCQ TYPE QUESTIONS

1992

1. A unity feedback system has the open loop transfer function

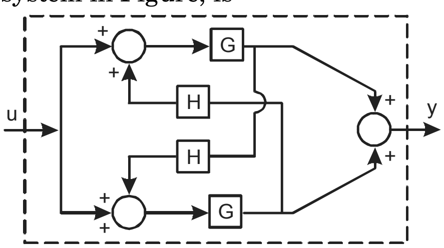
$$G(s) = \frac{1}{(s-1)(s+2)(s+3)}$$

The Nyquist plot of G encircles the origin

(*a*) Never (*b*) Once

(*c*) Twice (*d*) Thrice

2. The Nyquist plot encloses the origin only once from the above figure. Hence choice B is correct. The overall transfer function of the system in Figure, is

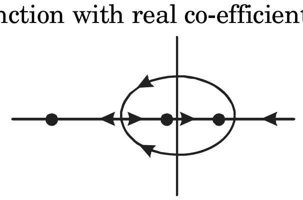


(*a*) $\frac{G}{1-GH}$ (*b*) $\frac{2G}{1-GH}$

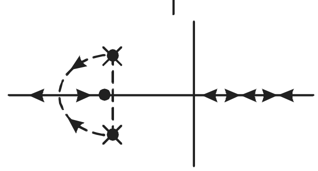
(*c*) $\frac{GH}{1-GH}$ (*d*) $\frac{2G}{1-H}$

3. Which of the following figure(s) represent valid root loci in the s-plane for positive K? Assume that the system has a transfer function with real co-efficients.

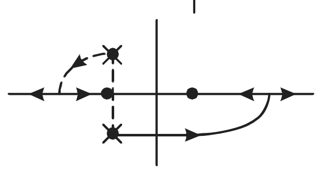
(*a*)



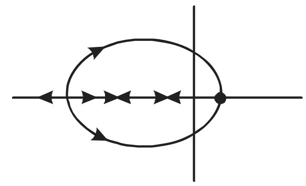
(*b*)



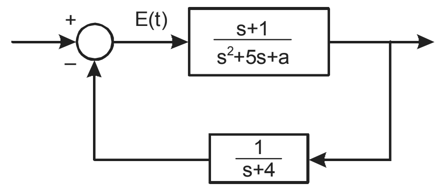
(*c*)



(*d*)



4. For what values of '*a*' does the system shown in Figure have a zero steady state error [i.e., $\lim_{t \to \infty} \in([t])$ for a step input ?



(*a*) $a = 0$

(*b*) $a = 0$

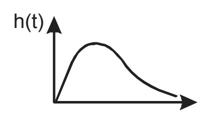
(*c*) $a \geq 4$

(*d*) for no value of '*a*'

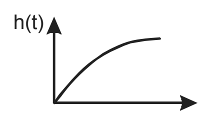
5. Match the following transfer functions and impulse responses

Transfer functions **Impulse Responses**

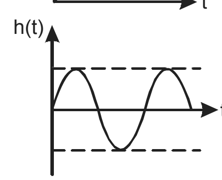
A. $\frac{s}{s+1}$ (P)



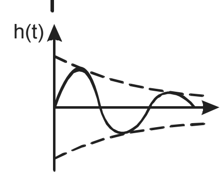
B. $\frac{1}{(s+1)^2}$ (Q)



C. $\frac{1}{s(s+1)+1}$ (R)



D. $\frac{1}{s^2+1}$ (S)



(*a*) A-P, B-Q, C-S, D-R

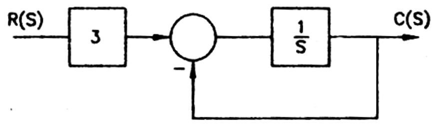
(*b*) A-Q, B-P, C-S, D-R

(*c*) A-P, B-Q, C-R, D-S

(*d*) A-P, B-S, C-Q, D-R

1994

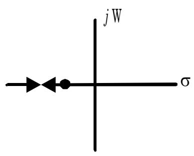
6. The matrix of any state–space equations for the transfer function $c(s)/R(s)$ of the system, shown below in Figure. is

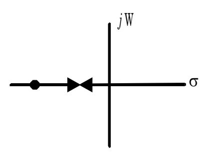


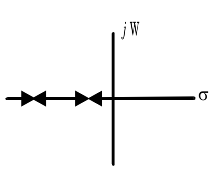
(a) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}$

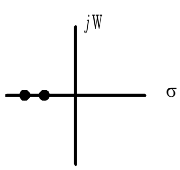
(c) [–1] (d) [3]

7. The pole–zero configuration of a phase–lead compensator is given by

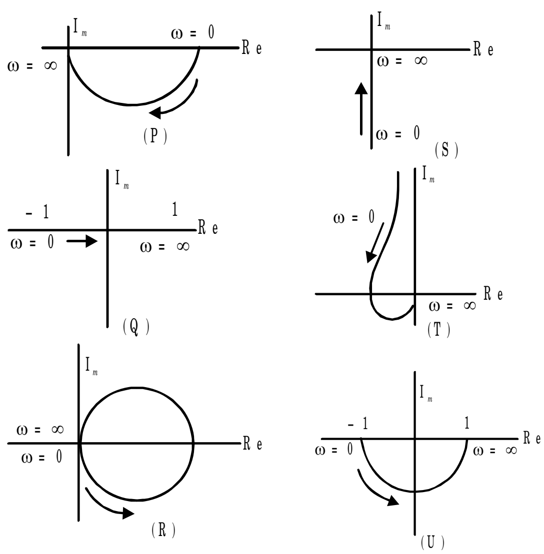
(a) 

(b) 

(c) 

(d) 

8. Match the polar plots for the following functions on the left hand side



(a) $\dfrac{s}{(s+1)(s+2)}$

(b) $\dfrac{s^2+1}{s^3}$

(c) $\dfrac{s^2-1}{s^2+1}$

(d) $\dfrac{1}{s^2+10}$

1995

9. The closed-loop transfer function of a control system is given by

$$\frac{C(s)}{R(s)} = \frac{2(s-1)}{(s+2)(s+1)}$$

For a unit step input the output is

(a) $-3e^{-2t} + 4e^{-t} - 1$ (b) $-3e^{-2t} - 4e^{-t} + 1$

(c) zero (d) infinity

10. A system is described by the state equation XX = AX + BU.

The output is given by Y = C X

where $A = \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ C = [1, 0].

Transfer function G(s) of the system is

(a) $\dfrac{s}{s^2+5s+7}$ (b) $\dfrac{1}{s^2+5s+7}$

(c) $\dfrac{s}{s^2+3s+2}$ (d) $\dfrac{1}{s^2+3s+2}$

1996

11. The unit-impulse response of a unit-feedback control system is given by

$$c(t) = -\,te^{-t} + 2\,e^{-t},\ (t \ge 0)$$

the open loop transfer function is equal to

(a) $\dfrac{s+1}{(s+2)^2}$ (b) $\dfrac{2s+1}{s^2}$

(c) $\dfrac{s+1}{(s+1)^2}$ (d) $\dfrac{s+1}{s^2}$

12. Consider the unit-step response of a unity-feedback control system whose open-loop transfer functions is $G(s) = \dfrac{1}{s(s+1)}$. The maximum overshoot is equal to

(a) 0.143 (b) 0.153

(c) 0.163 (d) 0.173

13. For a feedback control system of type 2, the steady state error for a ramp input is

(a) infinite (b) constant

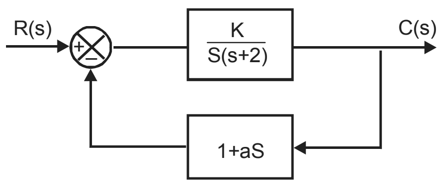
(c) zero (d) indeterminate

14. The closed-loop transfer function of a control system is given by $\dfrac{C(s)}{R(S)} = \dfrac{1}{1+s}$. For the input $r(t) = \sin t$, the steady state value of $c(t)$ is equal to

(a) $\dfrac{1}{\sqrt{2}}\cos t$ (b) 1

(c) $\dfrac{1}{\sqrt{2}}\sin t$ (d) $\dfrac{1}{\sqrt{2}}\sin\left(1 - \dfrac{\pi}{4}\right)$

15. For the system shown in Figure, with a damping ratio ζ of 0.7 and an undamped natural frequency ω_n of 4 rad/sec, the values of K and a are



(*a*) K = 4, *a* = 0.35 (*b*) K = 8, *a* = 0.455

(*c*) K = 16, *a* = 0.225 (*d*) K = 64, *a* = 0.9

16. The unit impulse response of a system is given as $c(t) = -4^{e-t} + 6e^{-2t}$ .The step response of the same system for $t \geq 0$ is equal to

(*a*) $-3e^{-2t} - 4e^{-t} + 1$

(*b*) $-3e^{-2t} + 4e^{-t} - 1$

(*c*) $-3e^{-2t} - 4e^{-t} - 1$

(*d*) $3e^{-2t} + 4e^{-t} - 1$

1997

17. Introduction of integral action in the forward path of a unity feedback system results in a

(*a*) marginally stable system

(*b*) system with no steady state error

(*c*) system with increased stability margin

(*d*) system with better speed of response

1998

18. The output of a linear time invariant control systme is $c(t)$ for a certain input $r(t)$. If $r(t)$ is modified by passing it through a block whose transfer function is e^{-s} and then applied to the systme, the modified output of the system would be

(*a*) $\dfrac{c(t)}{1+e^{t}}$ (*b*) $\dfrac{c(t)}{1+e^{-t}}$

(*c*) $c(t-1)\,u(t-1)$ (*d*) $c(t)\,u(t-1)$

19. None of the poles of a lonear control system lie in the right half of s-plane. For a bounded input, the output of this system

(*a*) is always bounded

(*b*) could be unbounded

(*c*) always tends to zero

(*d*) none of the above

20. The phase lead compensation is used to

(*a*) increase rise time and decrease overshoot

(*b*) decrease both rise time and overshoot

(*c*) increase both rise time and overshoot

(*d*) decrease rise time and increase overshoot

21. A set of linear equations is represented by the matrix equation Ax = b. The necessary condition for the existence of a solution for this system is

(*a*) A must be invertible

(*b*) *b* must be linearly depended on the columns of A

(*c*) *b* mut be linearly independent of the columns of A

(*d*) none of the above

22. The vector $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ is an eigen vector of A $= \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. One of the eigen values of A is

(*a*) 1 (*b*) 2

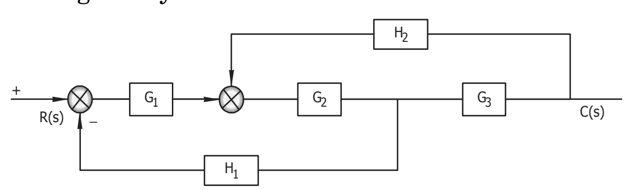
(*c*) 5 (*d*) – 1

23. A = $\begin{bmatrix} 2 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ -1 & 0 & 0 & 4 \end{bmatrix}$. The sum of the eigen values of the matrix A is

(*a*) 10 (*b*) – 10

(*c*) 24 (*d*) 22

24. For block diagram shown in Figure C(*s*)/R(*s*) is given by



(*a*) $\dfrac{G_1G_2G_3}{1+H_2G_2G_3+H_1G_1G_2}$

(*b*) $\dfrac{G_1G_2G_3}{1+G_1G_2G_3H_1H_2}$

(*c*) $\dfrac{G_1G_2G_3}{1+G_1G_2G_3H_1+G_1G_2G_3H_2}$

(*d*) $\dfrac{G_1G_2G_3}{1+G_1G_2G_3H_1}$

25. The number of roots on the equation $2S^4 + S^3 + 3S^2 + 5S + 7 = 0$ that lie in the right half of S plane is ;

(*a*) Zero

(*b*) One

(*c*) Two

(*d*) Three

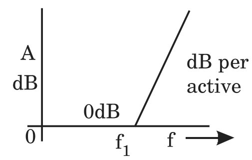
26. $A = \begin{bmatrix} 5 & 0 & 2 \\ 0 & 3 & 0 \\ 2 & 0 & 1 \end{bmatrix}$. The inverse of A is

(*a*) $\begin{bmatrix} 1 & 0 & -2 \\ 0 & \frac{1}{3} & 0 \\ -2 & 0 & 5 \end{bmatrix}$ (*b*) $\begin{bmatrix} 5 & 0 & 2 \\ 0 & -\frac{1}{3} & 0 \\ 2 & 0 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} \frac{1}{5} & 0 & \frac{1}{2} \\ 0 & \frac{1}{3} & 0 \\ \frac{1}{2} & 0 & 1 \end{bmatrix}$ (*d*) $\begin{bmatrix} \frac{1}{5} & 0 & -\frac{1}{2} \\ 0 & \frac{1}{3} & 0 \\ -\frac{1}{2} & 0 & 1 \end{bmatrix}$

1999

27. The function corresponding to the Bode plot of Figure, is



(*a*) $A = j\, f/f_1$ (*b*) $A = 1/(1 - j\, f_1/f)$

(*c*) $A = 1/(1 + jf_1/f)$ (*d*) $A = 1 + j\, f/f_1$

2000

28. Feedback control systems are

(*a*) insensitive to both forward-and feedback-path parameter changes

(*b*) less sensitive to feedback-path parameter changes than to forward-path parameter changes

(*c*) less sensitive to forward-path parameter changes than to feedback-path parameter changes

(*d*) equally sensitive to forward-and feed back- path parameter changes

29. A unity feedback system has open-loop transfer function G(*s*). The steady-state error is zero for

(*a*) step input and type-1G(*s*)

(*b*) ramp input and type-1G(*s*)

(*c*) step input and type-G(*s*)

(*d*) ramp input and type-0 G(*s*)

30. A linear time-invariant system initially at rest, when subjected to a unit-step input, gives a response $y(t) = te^{-t}, t > 0$. The transfer function of the system is

(*a*) $\frac{1}{(s+1)^2}$ (*b*) $\frac{1}{s(s+1)^2}$

(*c*) $\frac{s}{(s+1)^2}$ (*d*) $\frac{1}{s(s+1)}$

31. The characteristic equation of a feedback control system is

$$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$$

The number of roots in the right half of *s*-plane are

(*a*) zero (*b*) 1

(*c*) 2 (*d*) 3

32. A unity feedback system has open-loop transfer function $G(s) = \frac{25}{s(s+6)}$.

The peak overshoot in the step-input response of the system is approximately equal to

(*a*) 5% (*b*) 10%

(*c*) 15% (*d*) 20%

33. Maximum phase-lead of the compensator

$$D(s) = \frac{(0.5s + 1)}{(0.05s + 1)}, \text{ is}$$

(*a*) 52 deg at 4 rad/sec (*b*) 52 deg at 10 rad/sec

(*c*) 55 deg at 12 rad/sec (*d*) None of these

34. Open-loop transfer function of a unity-feedback system is

$$G(s) = G_1(s).\, e^{-r\tau_D} = \frac{e^{-s\tau_D}}{s(s+1)(s+2)}$$

Given : $|G_1(j\omega)| \approx 1$ when $\omega = 0.466$.

What is the phase margin when $\tau_D = 0$?

(*a*) 51.9° (*b*) 61.9°

(*c*) 41.9° (*d*) 71.9°

35. A unity feedback system has open-loop transfer function

$$G(s) = \frac{K(s+5)}{s(s+2)}\ ;\ K \geq 0$$

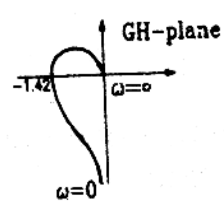
What is the value of K(if it exists) so that the damping ξ of the complex closed loop poles is 0.3?

(*a*) 1/2 (*b*) 5

(*c*) 2 (*d*) does not exist

2001

36. The polar plot of a type-1, 3-pole, open-loop system is shown in the figure given below. The closed-loop system is



(*a*) always stable

(*b*) marginally stable

(*c*) unstable with one pole on the right half s-plane

(*d*) unstable with two poles on the right half s-plane

37. Given the homogeneous state-space equation

$$\dot{x} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} x$$

The steady state value of $x_{ss} = \lim_{t \to \infty} x(t)$, given the initial state value of $x(0) = [10 - 10]^T$, is

(*a*) $x_{ss} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ (*b*) $x_{ss} = \begin{bmatrix} -3 \\ -2 \end{bmatrix}$

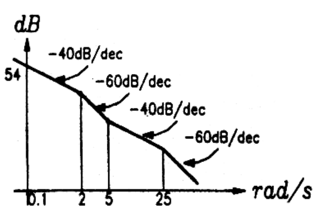
(*c*) $x_{ss} = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$ (*d*) $x_{ss} = \begin{bmatrix} \infty \\ \infty \end{bmatrix}$

38. The conductors of a 10 km long, single phase, two wire line are separated by a distance of 1.5 m. The diameter of each conductor is 1 cm. If the conductors are of copper, then inductance of the circuit is

(*a*) 50.0 mH (*b*) 45.3 mH

(*c*) 23.8 mH (*d*) 19.6 mH

39. The asymptotic approximation of the log-magnitude versus frequency plot of a minimum phase system with real poles and one zero is shown in the figure given below. Its transfer functions is



(*a*) $\dfrac{20(s+5)}{s(s+2)(s+25)}$ (*b*) $\dfrac{10(s+5)}{(s+2)^2(s+25)}$

(*c*) $\dfrac{20(s+5)}{s^2(s+2)\,(s+25)}$ (*d*) $\dfrac{50(s+5)}{s^2(s+2)\,(s+25)}$

Common Data Q. (40 – 42)

A unity feedback system has an open-loop transfer function of

$$G(s) = \frac{10000}{s(s+10)^2}$$

40. What is the magnitude of $G(j\omega)$ in *d*B at an angular frequency of $\omega = 20$ rad/sec?

(*a*) 0 dB (*b*) 10 dB

(*c*) 20 dB (*d*) none of these

41. What is the phase margin in degrees ?

(*a*) 36.86° (*b*) 56.86°

(*c*) –36.86° (*d*) none of these

42. What is the gain margin in dB ?

(*a*) –13.97 dB (*b*) 13.97 dB

(*c*) 43.97 dB (*d*) –43.97 dB

2002

43. Let s(t) be the step response of a linear system with zero initial conditions; then the response of this system to an input u(t) is

(*a*) $\int_0^t s(t-\tau)u(\tau)d\tau$

(*b*) $\dfrac{d}{dt}\left[\int_0^t s(t-\tau)u(\tau)d\tau\right]$

(*c*) $\int_0^t s(t-\tau)\left[\int_0^t u(\tau_1)d\tau_1\right]d\tau$

(*d*) $\int_0^1 s\,(t-\tau)^2\,u(\tau)\,d\tau$

44. Let Y(s) be the Laplace transformation of the function y(t), then final value of the function is

(*a*) $\lim_{s \to 0} Y(s)$ (*b*) $\lim_{s \to \infty} Y(s)$

(*c*) $\lim_{s \to 0} sY(s)$ (*d*) $\lim_{s \to \infty} sY(s)$

45. The determinant of the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 100 & 1 & 0 & 0 \\ 100 & 200 & 1 & 0 \\ 100 & 200 & 300 & 1 \end{bmatrix} \text{ is}$$

(*a*) 100 (*b*) 200

(*c*) 1 (*d*) 300

46. The state transition matrix for the system $\dot{X} = AX$ with initial state X(0) is

(*a*) $(sI - A)^{-1}$

(*b*) $e^{At}X(0)$

(*c*) Laplace inverse of $[(sI - A)^{-1}]$

(*d*) Laplace inverse of $[(sI - A)^{-1}X(0)]$

47. For the system $\dot{X} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$, which of the following statements is true ?

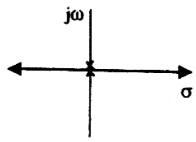
(*a*) The system is controllable but unstable

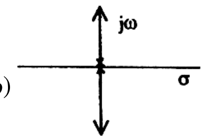
(*b*) The system is uncontrollable and unstable

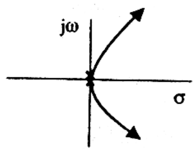
(*c*) The system is controllable and stable

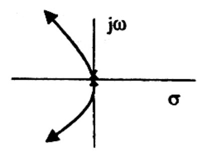
(*d*) The system is uncontrollable and stable

48. A unity feedback system has an open loop transfer function, $G(s) = \frac{K}{s^2}$. The root locus plot is

(*a*) 

(*b*) 

(*c*) 

(*d*) 

49. The transfer function of the system described by

$\frac{d^2y}{dt^2} + \frac{dy}{dt} = \frac{du}{dt} + 2u$ with u as input and y as output is

(*a*) $\frac{(s+2)}{(s^2+s)}$ (*b*) $\frac{(s+1)}{(s^2+s)}$

(*c*) $\frac{2}{(s^2+s)}$ (*d*) $\frac{2s}{(s^2+s)}$

50. For the system $\dot{X} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$; $y = \begin{bmatrix} 4 & 0 \end{bmatrix} X$, with u as unit impulse and with zero initial state, the output, y, becomes

(*a*) $2\,e^{2t}$ (*b*) $4\,e^{2t}$

(*c*) $2\,e^{4t}$ (*d*) $4\,e^{4t}$

51. The eigen values of the system represented by

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} X \text{ are}$$

(*a*) 0, 0, 0, 0 (*b*) 1, 1, 1, 1

(*c*) 0, 0, 0, –1 (*d*) 1, 0, 0,

2003

52. A control system is defined by the following mathematical relationship

$$\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 5x = 12\left(1 - e^{-2t}\right)$$

The response of the system as $t \to \infty$ is

(*a*) $x = 6$ (*b*) $x = 2$

(*c*) $x = 2.4$ (*d*) $x = -2$

53. A lead compensator used for a closed loop controller has the following transfer function

$$\frac{K\left(1 + \frac{s}{a}\right)}{\left(1 + \frac{s}{b}\right)}$$

For such a lead compensator

(*a*) $a < b$ (*b*) $b < a$

(*c*) $a > Kb$ (*d*) $a < Kb$

54. A second order system starts with an initial condition of $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$ without any external input. The state transition matrix for the system is given by $\begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}$. The state of the system at the end of 1 second is given by

(*a*) $\begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$ (*b*) $\begin{bmatrix} 0.135 \\ 0.368 \end{bmatrix}$

(*c*) $\begin{bmatrix} 0.271 \\ 0.736 \end{bmatrix}$ (*d*) $\begin{bmatrix} 0.135 \\ 1.100 \end{bmatrix}$

55. A control system with certain excitation is governed by the following mathematical equation

$$\frac{d^2x}{dt^2} + \frac{1}{2}\frac{dx}{dt} + \frac{1}{18}x = 10 + 5e^{-4t} + 2e^{-5t}$$

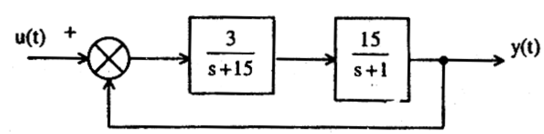
The natural time constants of the response of the system are

(*a*) 2s and 5s (*b*) 3s and 6s

(*c*) 4s and 5s (*d*) 1/3s and 1/6s

Common Data Q. (56 – 57)

The block diagram shown in the figure given below gives a unity feedback closed loop control system.



56. The steady state error in the response of the above system to unit step input is

(*a*) 25 % (*b*) 0.75 %

(*c*) 6 % (*d*) 33 %

57. The roots of the closed loop characteristic equation of the system are

(*a*) – 1 and – 15 (*b*) 6 and 10

(*c*) – 4 and – 15 (*d*) – 6 and – 10

58. The following equation defines a separately excited dc motor in the form of a differential equation

$$\frac{d^2\omega}{dt^2} + \frac{B}{J}\frac{d\omega}{dt} + \frac{K^2}{LJ}\omega = \frac{K}{LJ}V_a$$

The above equation may be organized in the state-space form as follows

$$\begin{bmatrix} \frac{d^2\omega}{dt^2} \\ \frac{d\omega}{dt} \end{bmatrix} = P\begin{bmatrix} \frac{d\omega}{dt} \\ \omega \end{bmatrix} + QV_a$$

where P matrix is given by

(*a*) $\begin{bmatrix} -\frac{B}{J} & -\frac{K^2}{LJ} \\ 1 & 0 \end{bmatrix}$

(*b*) $\begin{bmatrix} -\frac{K^2}{LJ} & -\frac{B}{J} \\ 0 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} 0 & 1 \\ -\frac{K^2}{LJ} & -\frac{B}{J} \end{bmatrix}$

(*d*) $\begin{bmatrix} 1 & 0 \\ -\frac{B}{J} & -\frac{K^2}{LJ} \end{bmatrix}$

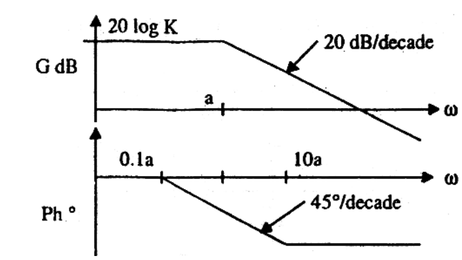
59. The loop gain GH of a closed loop system is given by the expression $\frac{K}{s(s+2)(s+4)}$

The value of K for which the system just becomes unstable is

(*a*) K = 6 (*b*) K = 8

(*c*) K = 48 (*d*) K = 96

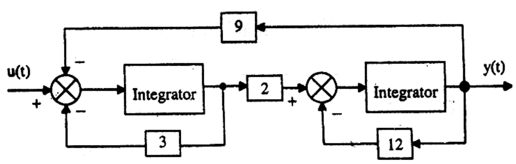
60. The asymptotic Bode plot of the transfer function $\frac{K}{1+\frac{s}{a}}$ is given in the figure given below. The error in phase angle and dB gain at a frequency of ω = 0.5 a are respectively



(*a*) 4.9°, 0.97 *d*B (*b*) 5.7°, 3 *d*B

(*c*) 4.9°, 3 *d*B (*d*) 5.7°, 0.97 *d*B

61. The block diagram of a control system is shown in the figure given below. The transfer function G(*s*) = Y(*s*)/U(*s*) of the system is



(*a*) $\frac{1}{18\left(1+\frac{s}{12}\right)\left(1+\frac{s}{3}\right)}$ (*b*) $\frac{1}{27\left(1+\frac{s}{6}\right)\left(1+\frac{s}{9}\right)}$

(*c*) $\frac{1}{27\left(1+\frac{s}{12}\right)\left(1+\frac{s}{9}\right)}$ (*d*) $\frac{1}{27\left(1+\frac{s}{9}\right)\left(1+\frac{s}{3}\right)}$

2004

62. The Nyquist plot of loop transfer function G(*s*) H(*s*) of a closed loop control system passes through the point (– 1, *j*0) in the G(*s*) H(*s*) plane. The phase margin of the system is

(*a*) 0° (*b*) 45°

(*c*) 90° (*d*) 180°

63. Consider the function, $F(s) = \frac{5}{s(s^2+3s+2)}$

where F(*s*) is Laplace transform of the function *f*(*t*). The initial value of *f*(*t*) is equal to

(*a*) 5 (*b*) $\frac{5}{2}$

(*c*) $\frac{5}{3}$ (*d*) 0

64. For a tachometer, if θ(*t*) is the rotor displacement is radians, *e*(*t*) is the output voltage and K_t is the tachometer constant in V/rad/sec, then the transfer function, $\frac{E(s)}{Q(s)}$ will be

(*a*) $K_t s^2$ (*b*) $\frac{K_t}{s}$

(*c*) $K_t s$ (*d*) K_t

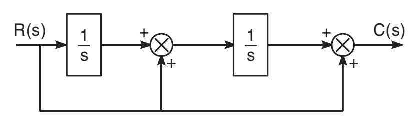
65. For the equation, $s^3 - 4s^2 + s + 6 = 0$

the number of roots in the left half of *s*-plane will be

(*a*) zero (*b*) one

(*c*) two (*d*) three

66. For the block diagram shown in the figure given below the transfer function $\frac{C(s)}{R(s)}$ is equal to



(*a*) $\frac{s^2+1}{s^2}$ (*b*) $\frac{s^2+s+1}{s^2}$

(*c*) $\frac{s^2+s+1}{s}$ (*d*) $\frac{1}{s^2+s+1}$

67. The state variable description of a linear autonomous system is,

$$X = AX,$$

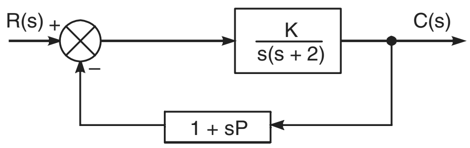
where X is two dimensional state vector, and A is the system matrix given by $A = \begin{bmatrix} 0 & 2 \\ 2 & 0 \end{bmatrix}$.

The roots of the characteristic equation are

(*a*) – 2 and + 2 (*b*) – *j*2 and + *j*2

(*c*) – 2 and – 2 (*d*) + 2 and + 2

68. The block diagram of a closed loop control system is given in the figure given below. The values of K and P such that the system has a damping ratio of 0.7 and an undamped natural frequency ω_n of 5 rad/sec, are respectively equal to



(*a*) 20 and 0.3 (*b*) 20 and 0.2
(*c*) 25 and 0.3 (*d*) 25 and 0.2

69. The unit impulse response of a second order under-damped system starting from rest is given by

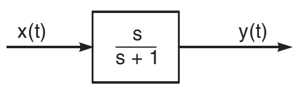
$$c(t) = 12.5\, e^{-6t} \sin 8\,t, \quad t \geq 0$$

The steady-state value of the unit step response of the system is equal to

(*a*) 0 (*b*) 0.25
(*c*) 0.5 (*d*) 1.0

70. In the system shown in the figure given below the input is $x(t) = \sin t$.

In the steady-state, the response $y(t)$ will be



(*a*) $\frac{1}{\sqrt{2}} \sin(t - 45°)$ (*b*) $\frac{1}{\sqrt{2}} \sin(t + 45°)$
(*c*) $\sin(t - 45°)$ (*d*) $\sin(t + 45°)$

71. The open loop transfer function of a unity feedback control system is given as $G(s) = \frac{as+1}{s^2}$. The value of 'a' to give a phase margin of 45° is equal to

(*a*) 0.141 (*b*) 0.441
(*c*) 0.841 (*d*) 1.141

2005

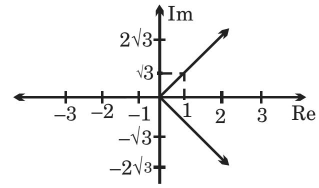
72. A system with zero initial conditions has the closed loop transfer function.

$$T(s) = \frac{s^2 + 4}{(s+1)(s+4)}.$$

The system output is zero at the frequency

(*a*) 0.5 rad/sec
(*b*) 1 rad/sec
(*c*) 2 rad/sec
(*d*) 4 rad/sec

73. Figure given below shows the root locus plot (location of poles not given) of a third order system whose open loop transfer function is



(*a*) $\frac{K}{s^3}$
(*b*) $\frac{K}{s^2(s+1)}$
(*c*) $\frac{K}{s(s^2+1)}$
(*d*) $\frac{K}{s(s^2-1)}$

74. The gain margin of a unity feedback control system with the open loop transfer function

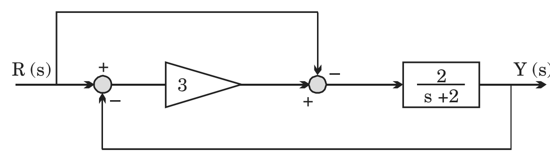
$$G(s) = \frac{(s+1)}{s^2} \text{ is}$$

(*a*) 0 (*b*) $\frac{1}{\sqrt{2}}$
(*c*) $\sqrt{2}$ (*d*) ∞

75. A unity feedback system, having an open loop gain $G(s)H(s) = \frac{K(1-s)}{(1+s)}$, becomes stable when

(*a*) $|K| > 1$ (*b*) $K > 1$
(*c*) $|K| < 1$ (*d*) $K < -1$

76. When subjected to a unit step input, the closed loop control system shown in the figure given below will have a steady state error of

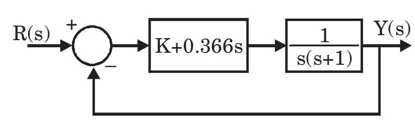


(*a*) – 1.0 (*b*) – 0.5
(*c*) 0 (*d*) 0.5

77. In the GH(s) plane, the Nyquist plot of the loop transfer function $G(s)H(s) = \frac{\pi e^{-0.25}}{s}$ passes through the negative real axis at the point

(*a*) $(-0.25, j0)$ (*b*) $(-0.5, j0)$
(*c*) $(-1, j0)$ (*d*) $(-2, j0)$

78. If the compensated system shown in the figure given below has a phase margin of 60° at the crossover frequency of 1 rad/sec, then value of the gain K is



(*a*) 0.366 (*b*) 0.732
(*c*) 1.366 (*d*) 2.738

79. For the matrix $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, one of the eigen values is equal to –2. Which of the following is an eigen vector ?

(a) $\begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$ (b) $\begin{bmatrix} -3 \\ 2 \\ -1 \end{bmatrix}$

(c) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ (d) $\begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$

80. If $u(t)$ is the unit step and $\delta(t)$ is the unit impulse function, the inverse z-transform of $F(z) = \frac{1}{z+1}$ for k > 0 is

(a) $(-1)^k \delta(k)$ (b) $\delta(k) - (-1)^k$

(c) $(-1)^k u(k)$ (d) $u(k) - (-1)^k$

Linked Answer Q. (81 – 82)

A state variable system

$$X(t) = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} X(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t),$$

with initial condition X(0) $[-1 \;\; 3]^T$ and the unit step input $u(t)$ has

81. The state transition matrix

(a) $\begin{bmatrix} 1 & \frac{1}{3}(1 - e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{3}(e^{-t} - e^{-3t}) \\ 0 & e^{-t} \end{bmatrix}$

(c) $\begin{bmatrix} 1 & \frac{1}{3}(e^{-t} - e^{-3t}) \\ 0 & e^{-3t} \end{bmatrix}$ (d) $\begin{bmatrix} 1 & (1 - e^{-t}) \\ 0 & e^{-t} \end{bmatrix}$

82. The state transition equation

(a) $X(t) = \begin{bmatrix} t - e^{-t} \\ e^{-t} \end{bmatrix}$

(b) $X(t) = \begin{bmatrix} t - e^{-3t} \\ e^{-t} \end{bmatrix}$

(c) $X(t) = \begin{bmatrix} t - e^{-3t} \\ 3e^{-3t} \end{bmatrix}$

(d) $X(t) = \begin{bmatrix} t - e^{-3t} \\ e^{-t} \end{bmatrix}$

2006

83. For a system with the transfer function $H(s) = \frac{3(s-2)}{4s^2 - 2s + 1}$, the matrix A in the state space form $x = Ax + Bu$ is equal to

(a) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 2 & -4 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & 1 & 0 \\ 3 & -2 & 1 \\ 1 & -2 & 4 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ -1 & 2 & -4 \end{bmatrix}$

84. A discrete real all pass system has a pole at $z = 2\angle 30°$: it, therefore

(a) also has a pole at $\frac{1}{2}\angle 30°$

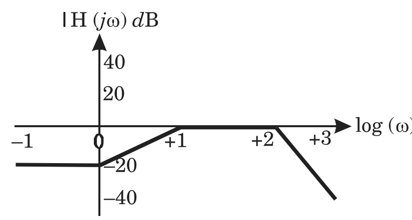
(b) has a constant phase response over the z-plane: arg $|H(z)|$ = constant

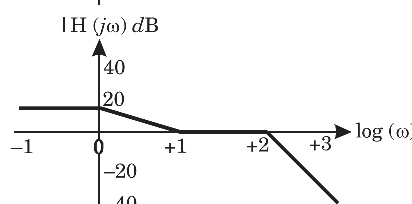
(c) is stable only, if it is anticausal

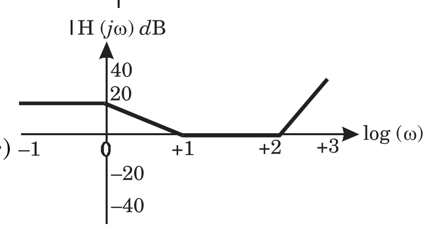
(d) has a constant phase response over the unit circle: arg $|H(e^{j\Omega})|$ = constant

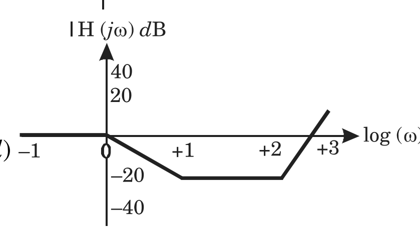
85. The Bode magnitude plot of

$$H(j\omega) = \frac{10^4(1 + j\omega)}{(10 + j\omega)(100 + j\omega)^2}$$ is

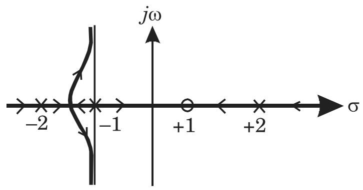
(a)


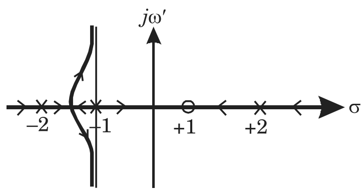
(b)


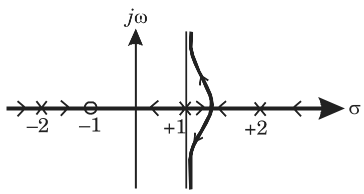
(c)


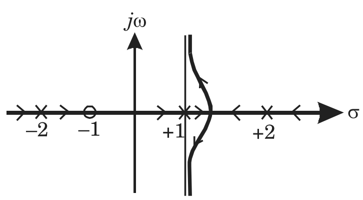
(d)


86. A closed-loop system has the characteristic function $(s^2 - 4)(s + 1) + K(s - 1) = 0$. Its root locus plot against K is

(*a*) 

(*b*) 

(*c*) 

(*d*) 

87. The algebraic equation

F(s) = $s^5 - 3s^4 + 5s^3 - 7s^2 + 4s + 20$ is given.

F(s) = 0 has

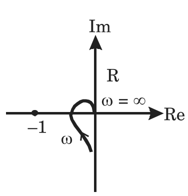
(*a*) a single complex root with the remaining roots being real

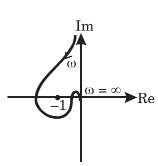
(*b*) one positive real root and four complex roots, all with positive real parts

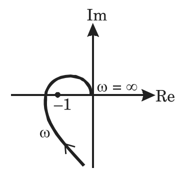
(*c*) one negative real root, two imaginary roots, and two roots with positive real parts

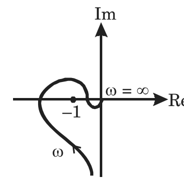
(*d*) one positive real root, two imaginary roots, and two roots with negative real parts

88. Consider the following Nyquist plots of loop transfer functions over ω = 0 to ω = ∞. Which of these plots represents a stable closed loop system ?

(1) 

(2) 

(3) 

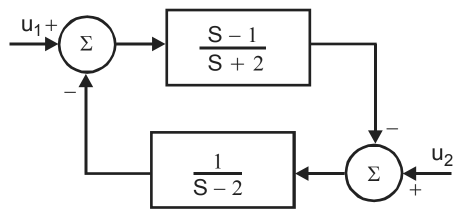
(4) 

(*a*) (1) only (*b*) all, except (1)

(*c*) all, except (3) (*d*) (1) and (2) only

2007

89. The system shown in the figure given below is



(*a*) stable

(*b*) unstable

(*c*) conditionally stable

(*d*) stable for input u_1, but unstable for input u_2

90. If $x = \text{Re}\, G(j\theta)$, and $y = \text{lm}\, G(j\omega)$ then for $\omega \to 0^+$, the Nyquist plot for $G(s) = \dfrac{1}{s(s+1)(s+2)}$ becomes asymptotic to the line

(*a*) $x = 0$ (*b*) $x = -\dfrac{3}{4}$

(*c*) $x = y - 1/6$ (*d*) $x = \dfrac{y}{\sqrt{3}}$

91. The system $\dfrac{900}{(s+1)(s+9)}$ is be compensated such that its gain-crossover frequency becomes same as its uncompensated phase-crossover frequency and provides a 45° phase margin. To achieve this, one may use

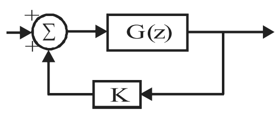
(*a*) a lag compensator that provides an attenuation of 20 dB and a phase lag of 45° at the frequency of $3\sqrt{3}$ rad/s

(*b*) a lead compensator that provides an amplification of 20 dB and a phase lead of 45° at the frequency of 3 rad/s

(*c*) a lag-lead compensator that provides an amplification of 20 dB and a phase lag of 45° at the frequency of $\sqrt{3}$ rad/s.

(*d*) a lag-lead compensator that provides an attenuation of 20 dB and phase lead of 45° at the frequency of 3 rad/s

92. Consider the discrete-time system shown in the figure where the impulse response of G(z) is $g(0) = 0, g(1) = g(2) = 1, g(3) = g(4) = \ldots = 0$.



This system is stable for range of values of K

(*a*) $\left[-1, \frac{1}{2}\right]$ (*b*) $[-1, 1]$

(*c*) $\left[-\frac{1}{2}, 1\right]$ (*d*) $\left[-\frac{1}{2}, 2\right]$

93. If the loop gain K of a negative feedback system having a loop transfer function $\frac{K(s+3)}{(s+8)^2}$ is to be adjusted to induce a sustained oscillation then

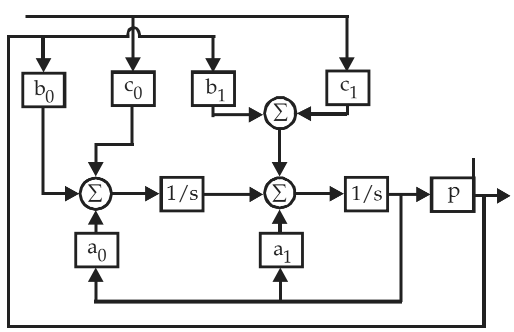
(*a*) The frequency of this oscillation must be $\frac{4}{\sqrt{3}}$ rad/s

(*b*) The frequency of this oscillation must be must be 4 rad/s

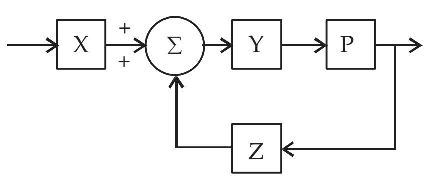
(*c*) The frequency of this oscillation must be must be 4 or $\frac{4}{\sqrt{3}}$ rad/s

(*d*) such a K does not exist

94. The system shown in the figure below



can be reduced to the form



with

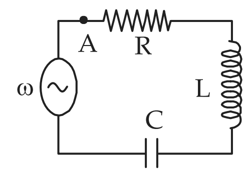
(*a*) $X = c_0s + c_1,\ Y = \frac{1}{(s^2 + a_0s + a_1)},\ Z = b_0s + b_1$

(*b*) $X = 1,\ Y = \frac{(c_0s + c_1)}{(s^2 + a_0s + a_1)},\ Z = b_0s + b_1$

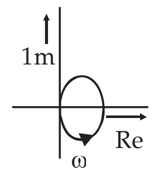
(*c*) $X = c_1s + c_0,\ Y = \frac{(b_1s + b_0)}{(s^2 + a_1s + a_0)},\ Z = 1$

(*d*) $X = c_1s + c_0,\ Y = \frac{1}{(s^2 + a_1s + a_0)},\ Z = b_1s + b_0$

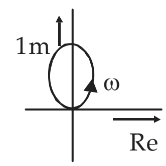
95. The R-L-C series circuit shown is supplied from a variable frequency voltage source. The admittance - locus of the R-L-C network at terminals AB for increasing frequency ω is



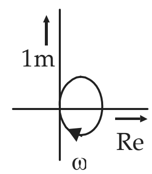
(*a*)



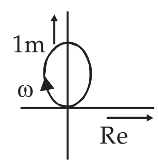
(*b*)



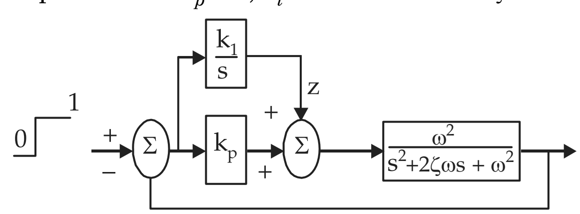
(*c*)



(*d*)



96. Consider the feedback control system shown below which is subjected to a unit step input. The system is stable and has the following parameters $k_p = 4$, $k_i = 10$ $\omega = 500$ and $\xi = 0.7$



The steady state value of z is

(*a*) 1 (*b*) 0.25

(*c*) 0.1 (*d*) 0

2008

97. The characteristic equation of a (3 × 3) matrix P is defined as

$$\alpha(\lambda) = |\lambda I - P| = \lambda^3 + \lambda^2 + 2\lambda + 1 = 0.$$

If I denotes identity matrix, then the inverse of matrix P will be

(*a*) $(P^2 + P + 2I)$

(*b*) $(P^2 + P + I)$

(*c*) $-(P^2 + P + I)$

(*d*) $-(P^2 + P + 2I)$

98. The transfer function of a linear time invariant system is given as

$$G(s) = \frac{1}{s^2 + 3s + 2}$$

The steady state value of the output of this system for a unit impulse input applied at time instant t = 1 will be

(*a*) 0 (*b*) 0.5

(*c*) 1 (*d*) 2

99. The transfer functions of two compensators are

$$C_1 = \frac{10(s+1)}{(s+10)},\ C_2 = \frac{s+10}{10(s+1)}$$

Which one of the following statements is correct?

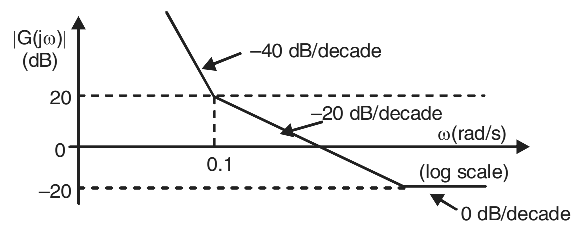
(*a*) C_1 is a lead compensator and C_2 is a lag compensator

(*b*) C_1 is a lag compensator and C_2 is a lead compensator

(*c*) Both C_1 and C_2 are lead compensators

(*d*) Both C_1 and C_2 are lag compensators

100. The asymptotic Bode magnitude plot of a minimum phase transfer function is shown in the figure



This transfer function has

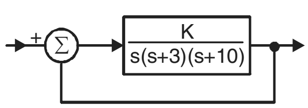
(*a*) Three poles and one zero

(*b*) Two poles and one zero

(*c*) Two poles and two zeros

(*d*) One pole and two zeros

101. Figure shows a feedback system where K > 0. The range of K for which the system is stable will be given by



(*a*) 0 < K < 30 (*b*) 0 < K < 39

(*c*) 0 < K < 390 (*d*) K > 390

102. The transfer function of a system is given as

$$\frac{100}{s^2 + 20s + 100}$$

This system is

(*a*) an overdamped system

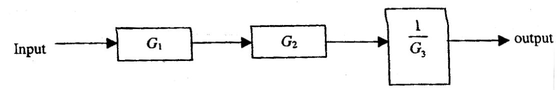
(*b*) an underdamped system

(*c*) a critically damped system

(*d*) an unstable system

2009

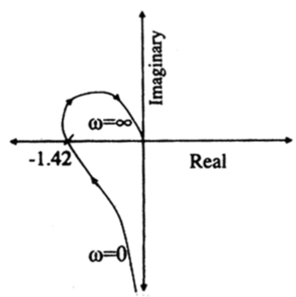
103. The measurement system shown in the figure uses three sub-systems in cascade whose gains are specified as G_1, G_2 and $\frac{1}{G_3}$. The relative small errors associated with each respective subsystem G_1, G_2 and G_3 are ε_1, ε_2 and ε_3. The error associated with the output is



(*a*) $\varepsilon_1 + \varepsilon_2 + \frac{1}{\varepsilon_3}$ (*b*) $\frac{\varepsilon_1 . \varepsilon_2}{\varepsilon_3}$

(*c*) $\varepsilon_1 + \varepsilon_2 - \varepsilon_3$ (*d*) $\varepsilon_1 + \varepsilon_2 + \varepsilon_3$

104. The polar plot of an open loop stable system is shown below. The closed loop system is



(*a*) always stable

(*b*) marginally stable

(*c*) unstable with one pole on the RH s-plane

(*d*) unstable with two poles on the RH s-plane

105. The first two rows of Routh's tabulation of a third order equation are as follows.

s^3	2	2
s^3	4	4

This means there are

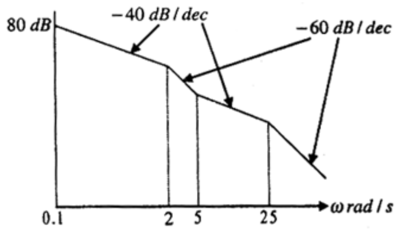
(*a*) two roots at $s = \pm j$ and one root in right half s-plane

(*b*) two roots at $s = \pm j2$ and one root in left half s-plane

(*c*) two roots at $s = \pm j2$ and one root in right half s-plane

(*d*) two roots at $s = \pm j$ and one root in left half s-plane

106. The asymptotic approximation of the log-magnitude *vs* frequency plot of a system containing only real poles and zeros is shown in the figure given below. Its transfer function is

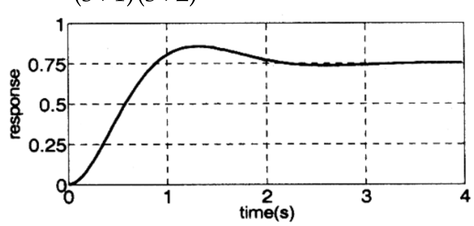


(*a*) $\dfrac{10(s+5)}{s(s+2)(s+25)}$ (*b*) $\dfrac{1000(s+5)}{s^2(s+2)(s+25)}$

(*c*) $\dfrac{100(s+5)}{s(s+2)(s+25)}$ (*d*) $\dfrac{80(s+5)}{s^2(s+2)(s+25)}$

107. The unit-step response of a unity feedback system with open loop transfer function

$G(s) = \dfrac{K}{(s+1)(s+2)}$ is shown in the figure below.



The value of *K* is

(*a*) 0.5 (*b*) 2

(*c*) 4 (*d*) 6

108. The open loop transfer function of a unity feedback system is given by

$$G(s) = \frac{(e^{-0.1s})}{s}.$$

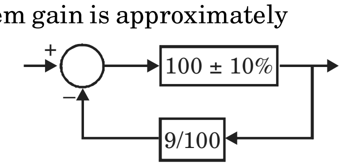
The gain margin of this system is

(*a*) 11.95 dB (*b*) 17.67 dB

(*c*) 21.33 dB (*d*) – 23.9 dB

2010

109. As shown in the figure, a negative feedback system has an amplifier of gain 100 with ±10 % tolerance in the forward path, and an attenuator of value $\dfrac{9}{100}$ in the f eedback path. The overall system gain is approximately



(*a*) 10 ± 1% (*b*) 10 ± 2%

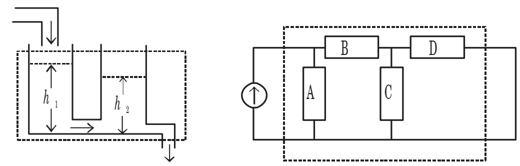
(*c*) 10 ± 5% (*d*) 10 ± 10%

110. For the system $\dfrac{2}{(s+1)}$, the approximate time taken for a step response to reach 98% of its final value is

(*a*) 1 s (*b*) 2 s

(*c*) 4 s (*d*) 8 s

111. If the electrical circuit of Fig.(*b*) is an equivalent of the coupled tank system of Fig.(*a*), then



(*a*) Coupled tank (*b*) Electrical equivalent

(*a*) A, B are resistances and C, D capacitances

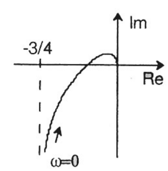
(*b*) A, C are resistances and B, D capacitances

(*c*) A, B are capacitances and C, D resistances

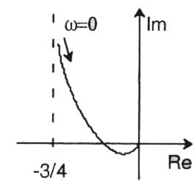
(*d*) A, C are capacitances and B, D resistances

112. The frequency response of $G(s) = \dfrac{1}{[s(s+1)(s+2)]}$ plotted in the complex $G(j\omega)$ plane (for $0 < \omega < \infty$) is

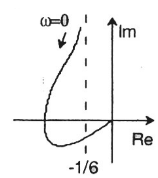
(*a*)



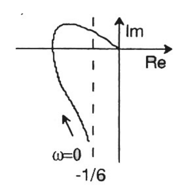
(*b*)



(*c*)



(*d*)



113. The system $\dot{x} = Ax + Bu$ with $A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ is

(*a*) stable and controllable

(*b*) stable but uncontrollable

(*c*) unstable but controllable

(*d*) unstable and uncontrollable

114. The characteristic equation of a closed-loop system is $s(s+1)(s+3) + k(s+2) = 0, k > 0$. Which of the following statements is true?

(*a*) Its roots are always real

(*b*) It cannot have a breakaway point in the range $-1 < \text{Re}[s] < 0$

(*c*) Two of its roots tend to infinity along the asymptotes $\text{Re}[s] = -1$

(*d*) It may have complex roots in the right half plane

2011

115. The frequency response of a linear system $G(j\omega)$ is provided in the tabular form below

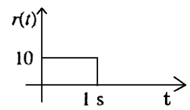
$\lvert G(j\omega)\rvert$	1.3	1.2	1.0	0.8	0.5	0.3
$\angle G(j\omega)$	$-130°$	$-140°$	$-150°$	$-160°$	$-180°$	$-200°$

The *gain margin* and *phase margin* of the system are

(*a*) 6 dB and 30° (*b*) 6 dB and – 30°

(*c*) – 6 dB and 30° (*d*) – 6 dB and – 30°

116. The steady state error of a unity feedback linear system for a unit step input is 0.1. The steady state error of the same system, for a pulse input $r(t)$ having a magnitude of 10 and a duration of one second, as shown in the figure is



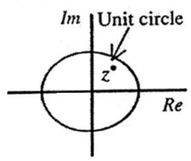
(*a*) 0

(*b*) 0.1

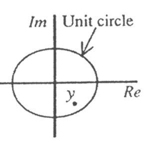
(*c*) 1

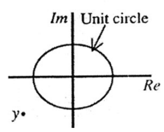
(*d*) 10

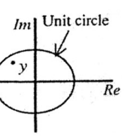
117. A point z has been plotted in the complex plane, as shown in figure below.

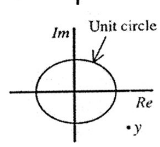


The plot of the complex number $y = \frac{1}{z}$ is

(*a*)


(*b*)


(*c*)


(*d*)


118. An open loop system represented by the transfer function $G(s) = \frac{(s-1)}{(s+2)(s+3)}$ is

(*a*) stable and of the minimum phase type

(*b*) stable and of the non-minimum phase type

(*c*) unstable and of the minimum phase type

(*d*) unstable and of the non-minimum phase type

119. Let the Laplace transform of a function $f(t)$ which exists for $t > 0$ be $F_1(s)$ and the Laplace transform of its delayed version $f(t-\tau)$ be $F_2^*(s)$. Let $F_1^*(s)$ be the complex conjugate of $F_1(s)$ with the Laplace variable set as $s = \sigma + j\omega$. If $G(s) = \frac{F_2(s).F_1^*(s)}{|F_1(s)|^2}$, then the inverse Laplace transform of $G(s)$ is

(*a*) an ideal impulse $\delta(t)$

(*b*) an ideal delayed impulse $\delta(t-\tau)$

(*c*) an ideal step function $u(t)$

(*d*) an ideal delayed step function impulse $u(t-\tau)$

120. The open loop transfer function $G(s)$ of a unity feedback control system is given as,

$$G(s) = \frac{k\left(s+\frac{2}{3}\right)}{s^2(s+2)}$$

From the root locus, it can be inferred that when k tends to positive infinity,

(*a*) three roots with nearly equal real parts exist on the left half of the s-plane

(*b*) one real root is found on the right half of s-plane

(*c*) the root loci cross the $j\omega$ axis for a finite value of k; $k \neq 0$

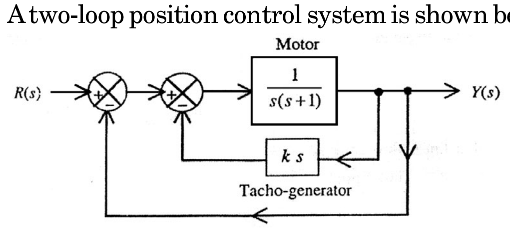
(*d*) three real roots are found on the right half of the s-plane

121. The matrix $[A] = \begin{bmatrix} 2 & 1 \\ 4 & -1 \end{bmatrix}$ is decomposed into a product of a lower triangular matrix [L] and an upper triangular matrix [U]. The properly decomposed [L] and [U] matrices respectively are

(*a*) $\begin{bmatrix} 1 & 0 \\ 4 & -1 \end{bmatrix}$ & $\begin{bmatrix} 1 & 1 \\ 0 & -2 \end{bmatrix}$ (*b*) $\begin{bmatrix} 2 & 0 \\ 4 & -1 \end{bmatrix}$ & $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

(*c*) $\begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}$ & $\begin{bmatrix} 2 & 1 \\ 0 & -1 \end{bmatrix}$ (*d*) $\begin{bmatrix} 2 & 0 \\ 4 & -3 \end{bmatrix}$ & $\begin{bmatrix} 1 & 0.5 \\ 0 & 1 \end{bmatrix}$

122. A two-loop position control system is shown below.



The gain k of the Tacho-generator influences mainly

(*a*) peak overshoot

(*b*) natural frequency of oscillation

(*c*) phase shift of the closed loop transfer function at very low frequencies ($\omega \to 0$)

(*d*) phase shift of the closed loop transfer function at very high frequencies ($\omega \to \infty$)

2012

123. A system with transfer function

$$G(s) = \frac{(s^2+9)(s+2)}{(s+1)(s+3)(s+4)}$$

is excited by sin (ωt). The steady-state output of the system is zero at

(*a*) $\omega = 1$ rad/s (*b*) $\omega = 2$ rad/s

(*c*) $\omega = 3$ rad/s (*d*) $\omega = 4$ rad/s

124. The unilateral Laplace transform of $f(t)$ is $\dfrac{1}{s^2+s+1}$.

The unilateral Laplace transform of $t\ f(t)$ is

(*a*) $-\dfrac{s}{(s^2+s+1)^2}$ (*b*) $-\dfrac{2s+1}{(s^2+s+1)^2}$

(*c*) $\dfrac{s}{(s^2+s+1)^2}$ (*d*) $\dfrac{2s+1}{(s^2+s+1)^2}$

125. The state variable description of an LTI system is given by

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{pmatrix}=\begin{pmatrix}0 & a_1 & 0\\ 0 & 0 & a_1\\ a_3 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}\begin{pmatrix}0\\ 0\\ 1\end{pmatrix}u,\ y=(1\ 0\ 0)\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

where y is the output and u is the input. The system is controllable for

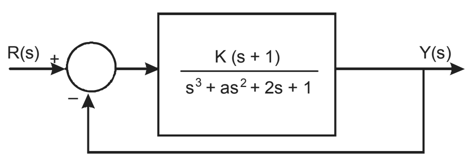
(*a*) $a_1 \neq 0, a_2 = 0, a_3 \neq 0$

(*b*) $a_1 = 0, a_2 \neq 0, a_3 \neq 0$

(*c*) $a_1 = 0, a_2 \neq 0, a_3 = 0$

(*d*) $a_1 \neq 0, a_2 \neq 0, a_3 = 0$

126. The feedback system shown below oscillates at 2 rad/s when



(*a*) K = 2 and a = 0.75 (*b*) K = 3 and a = 0.75

(*c*) K = 4 and a = 0.5   (*d*) K = 2 and a = 0.5

127. The input $x(t)$ and output $y(t)$ of a system are related as $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)d\tau$. The system is

(*a*) time-invariant and stable

(*b*) stable and not time-invariant

(*c*) time-invariant and not stable

(*d*) not time-invariant and not stable

2013

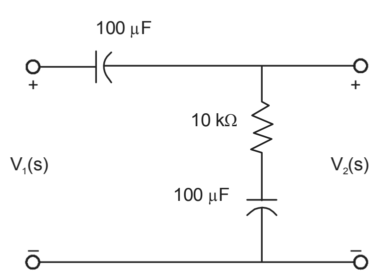
128. The transfer function $\dfrac{V_2(s)}{V_1(s)}$ of the circuit shown below is

(*a*) $\dfrac{0.5s+1}{s+1}$

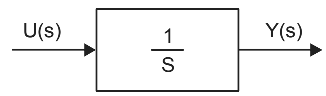
(*b*) $\dfrac{3s+6}{s+2}$

(*c*) $\dfrac{s+2}{s+1}$

(*d*) $\dfrac{s+1}{s+2}$



129. Assuming zero initial condition, the response $y(t)$ of the system given below to a unit step input $u(t)$ is



(*a*) $u(t)$ (*b*) $t\ u(t)$

(*c*) $\dfrac{t^2}{2}u(t)$ (*d*) $e^{-1}u(t)$

130. The impulse response of a system is $h(t) = t\ u(t)$. For an input $u(t-1)$, the output is

(*a*) $\dfrac{t^2}{2}u(t)$ (*b*) $\dfrac{t(t-1)}{2}u(t-1)$

(*c*) $\dfrac{(t-1)^2}{2}u(t-1)$ (*d*) $\dfrac{t^2-1}{2}u(t-1)$

131. Which one of the following statements is NOT TRUE for a continuous time causal and stable LTI system?

(*a*) All the poles of the system must lie on the left side of the $j\omega$ axis.

(*b*) Zeros of the system can lie anywhere in the s-plane.

(*c*) All the poles must lie within $|s| = 1$.

(*d*) All the roots of the characteristic equation must be located on the left side of the $j\omega$ axis.

132. Two systems with impulse responses $h_1(t)$ and $h_2(t)$ are connected in cascade. Then the overall impulse response of the cascaded system is given by

(*a*) product of $h_1(t)$ and $h_2(t)$

(*b*) sum of $h_1(t)$ and $h_2(t)$

(*c*) convolution of $h_1(t)$ and $h_2(t)$

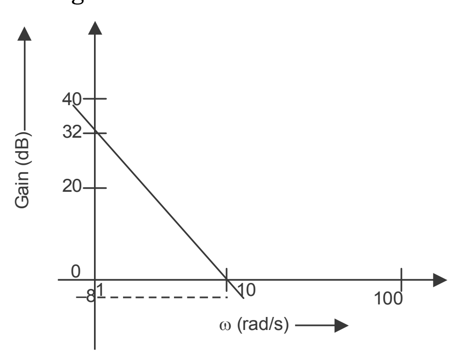
(*d*) subtraction of $h_2(t)$ from $h_1(t)$

133. A source $v_s(t) = V\cos 100\pi t$ has an internal impedance of $(4 + j3)\Omega$. If a purely resistive load connected to this source has to extract the maximum power out of the source, its value in Ω should be

(*a*) 3   (*b*) 4

(*c*) 5   (*d*) 7

134. The Bode plot of a transfer function G (s) is shown in the figure below.

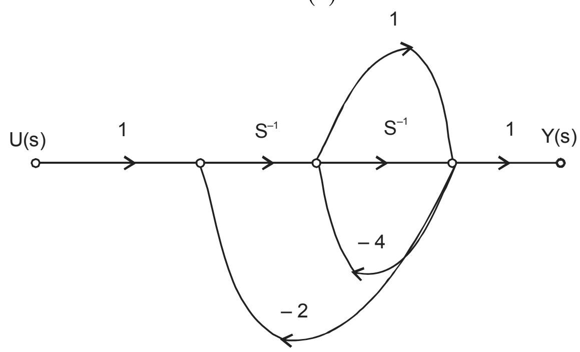


The gain (20 log | G(s) |) is 32 dB and –8 dB at 1 rad/s and 10 rad/s respectively. The phase is negative for all ω. Then G(s) is

(a) $\frac{39.8}{s}$ (b) $\frac{39.8}{s^2}$

(c) $\frac{32}{s}$ (d) $\frac{32}{s^2}$

135. The signal flow graph for a system is given below. The transfer function $\frac{Y(s)}{U(s)}$ for this system is



(a) $\frac{s+1}{5s^2+6s+2}$ (b) $\frac{s+1}{s^2+6s+2}$

(c) $\frac{s+1}{s^2+4s+2}$ (d) $\frac{1}{5s^2+6s+2}$

136. The impulse response of a continuous time system is given by $h(t) = \delta(t-1) + \delta(t-3)$. The value of the step response at $t = 2$ is

(a) 0 (b) 1

(c) 2 (d) 3

2014

137. In the formation of Routh-Hurwitz array for a polynomial, all the elements of a row have zero values. This premature termination of the array indicates the presence of

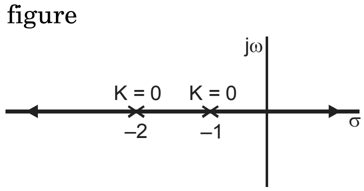
(a) Only one root at the origin

(b) Imaginary roots

(c) Only positive real roots

(d) Only negative real roots

138. The root locus of a unity feedback system is shown in the figure



The closed loop transfer function of the system is

(a) $\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)}$

(b) $\frac{C(s)}{R(s)} = \frac{-K}{(s+1)(s+2)+K}$

(c) $\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)-K}$

(d) $\frac{C(s)}{R(s)} = \frac{K}{(s+1)(s+2)+K}$

139. The state transition matrix for the system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}1\\ 1\end{bmatrix}u \text{ is}$$

(a) $\begin{bmatrix}e^t & 0\\ e^t & e^t\end{bmatrix}$ (b) $\begin{bmatrix}e^t & 0\\ t^2e^t & e^t\end{bmatrix}$

(c) $\begin{bmatrix}e^t & 0\\ te^t & e^t\end{bmatrix}$ (d) $\begin{bmatrix}e^t & te^t\\ 0 & e^t\end{bmatrix}$

140. For the transfer function

$$G(s) = \frac{5(s+4)}{s(s+0.25)(s^2+4s+25)}$$

The values of the constant gain term and the highest corner frequency of the Bode plot respectively are

(a) 3.2, 5.0 (b) 16.0, 4.0

(c) 3.2, 4.0 (d) 16.0, 5.0

141. The second order dynamic system

$$\frac{dX}{dt} = PX + Qu \qquad y = RX$$

has the matrices P, Q and R as follows:

$$P = \begin{bmatrix}-1 & 1\\ 0 & -3\end{bmatrix} \quad Q = \begin{bmatrix}0\\ 1\end{bmatrix} \quad R = [0 \;\; 1]$$

The system has the following controllability and observability properties:

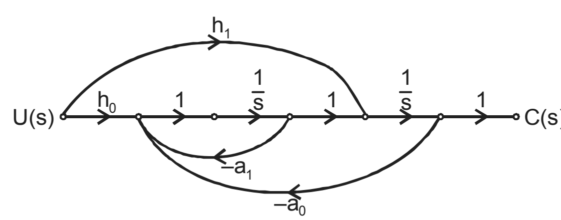
(a) Controllable and observable

(b) Not controllable but observable

(c) Controllable but not observable

(d) Not controllable and not observable

142. The signal flow graph of a system is shown below. U(s) is the input and C(s) is the output



Assuming, $h_1 = b_1$ and $h_0 = b_0 - b_1 a_1$, then input-output transfer function, $G(s) = \frac{C(s)}{U(s)}$ of the system is given by (*a*) $G(s) = \dfrac{b_0 s + b_1}{s^2 + a_0 s + a_1}$ (*b*) $G(s) = \dfrac{a_1 s + a_0}{s^2 + b_1 s + b_0}$

(*c*) $G(s) = \dfrac{b_1 s + b_0}{s^2 + a_1 s + a_0}$ (*d*) $G(s) = \dfrac{a_0 s + a_1}{s^2 + b_0 s + b_1}$

143. A single-input single-output feedback system has forward transfer function G(s) and feedback transfer function H(s). It is given that $|G(s).H(s)| < 1$. Which of the following is true about the stability of the system?

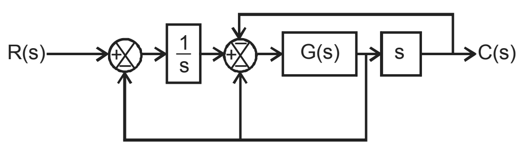
(*a*) The system is always stable

(*b*) The system is stable if all zeros of G(s).H(s) are in left half of the s-plane

(*c*) The system is stable if all poles of G(s).H(s) are in left half of the s-plane

(*d*) It is not possible to say whether or not the system is stable from the information given

144. The block diagram of a system is shown in the figure



If the desired transfer function of the system is

$$\frac{C(s)}{R(s)} = \frac{s}{s^2 + s + 1}$$

Then G(s) is

(*a*) 1 (*b*) s

(*c*) $1/s$ (*d*) $\dfrac{-s}{s^3 + s^2 - s - 2}$

145. Consider the system described by following state space equations

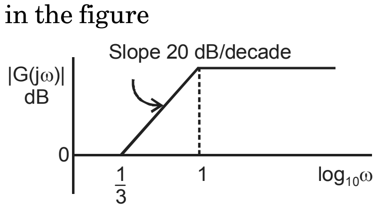
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \; y = [1 \; 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

If u is unit step input, then the steady state error of the system is

(*a*) 0 (*b*) $\dfrac{1}{2}$

(*c*) $\dfrac{2}{3}$ (*d*) 1

146. The magnitude Bode plot of a network is shown in the figure



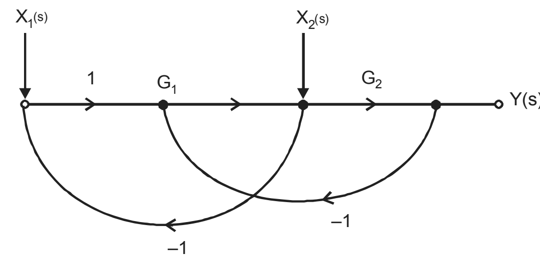
The maximum phase angle ϕ_m and the corresponding gain G_m respectively, are

(*a*) –30° and 1.73dB (*b*) –30° and 4.77dB

(*c*) +30° and 4.77dB (*d*) +30° and 1.73dB

2015

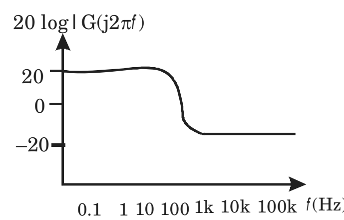
147. For the signal–flow graph shown in the figure, which one of the following expressions is equal to the transfer function $\left.\dfrac{Y(s)}{X_2(s)}\right|_{X_1(s)=0}$.



(*a*) $\dfrac{G_1}{1 + G_2(1 + G_1)}$ (*b*) $\dfrac{G_2}{1 + G_1(1 + G_2)}$

(*c*) $\dfrac{G_1}{1 + G_1 G_2}$ (*d*) $\dfrac{G_2}{1 + G_1 G_2}$

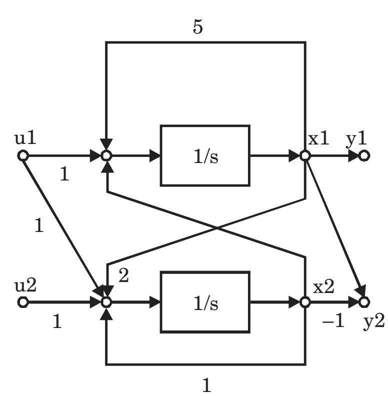
148. A Bode magnitude plot for the transfer function G(s) of a plant is shown in the figure. Which one of the following transfer functions best describes the plant?



(*a*) $\dfrac{1000(s + 10)}{s + 1000}$ (*b*) $\dfrac{10(s + 10)}{s(s + 1000)}$

(*c*) $\dfrac{s + 1000}{10s(s + 10)}$ (*d*) $\dfrac{s + 1000}{10(s + 10)}$

149. In the signal flow diagram given in the figure, u_1 and u_2 are possible inputs whereas y_1 and y_2 are possible outputs.When would the SISO system derived from this diagram be controlable and observable ?



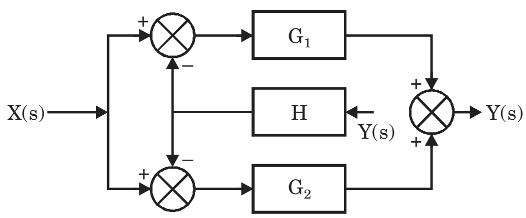
(*a*) When u_1 is the only input and y_1 is the only output

(*b*) When u_2 is the only input and y_1 is the only output

(*c*) When u_1 is the only input and y_2 is the only output

(*d*) When u_2 is the only input and y_2 is the only output

150. Find the transfer function $\frac{Y(s)}{X(s)}$ of the system given below.



(*a*) $\frac{G_1}{1-HG_1}+\frac{G_2}{1-HG_2}$ (*b*) $\frac{G_1}{1+HG_1}+\frac{G_2}{1+HG_2}$

(*c*) $\frac{G_1+G_2}{1+H(G_1+G_2)}$ (*d*) $\frac{G_1+G_2}{1-H(G_1+G_2)}$

151. The transfer function of a second order real system with a perfectly flat magnitude response of unity has a pole at (2 – j3). List all the poles and zeroes.

(*a*) Poles at (2 ± j3), no zeroes

(*b*) Poles at (±2 – j3), one zero at origin

(*c*) Poles at (2 – j3), (– 2 + j3), zeroes at (– 2 – j3), (2 + j3)

(*d*) Poles at (2 ± j3), zeroes at (–2 ± j3)

152. The open loop poles of a third order unity feedback system are at 0, –1, –2. Let the frequency corresponding to the point where the root locus of the system transits to unstable region be K. Now suppose we introduce a zero in the open loop transfer function at –3, while keeping all the earlier open loop poles intact. Which one of the following is TRUE about the point where the root locus of the modified system transits to unstable region?

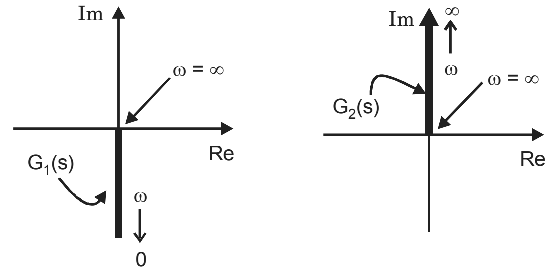
(*a*) It corresponds to a frequency greater than K

(*b*) It corresponds to a frequency less than K

(*c*) It corresponds to a frequency K

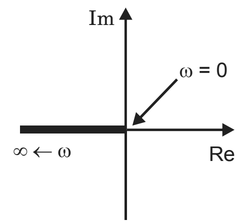
(*d*) Root locus of modified system never transits to unstable region

153. Nyquist plot of the functions $G_1(s)$ and $G_2(s)$ are shown in figure.

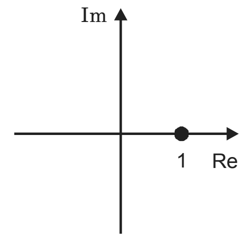


Nyquist plot of the product of $G_1(s)$ and $G_2(s)$ is

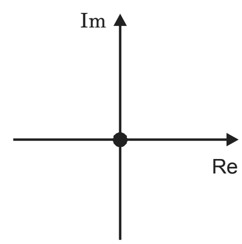
(*a*)



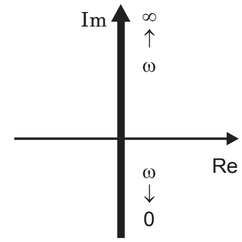
(*b*)



(*c*)



(*d*)



154. An open loop transfer function G(*s*) of a system is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

For a unity feedback system, the breakaway point of the root loci on the real axis occurs at,

(*a*) – 0.42 (*b*) – 1.58

(*c*) – 0.42 and –1.58 (*d*) None of the above

2016

155. The transfer function of a system is $\frac{Y(s)}{R(s)} = \frac{s}{s+2}$.

The steady state output $y(t)$ is A cos(2t + ϕ) for the input cos(2t). The values of A and ϕ, respectively are

(*a*) $\frac{1}{\sqrt{2}}, -45°$ (*b*) $\frac{1}{\sqrt{2}}, +45°$

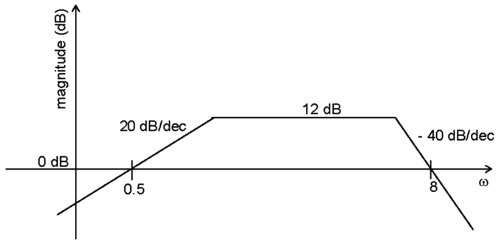
(*c*) $\sqrt{2}, -45°$ (*d*) $\sqrt{2}, +45°$

156. The phase cross-over frequency of the transfer function $G(s) = \frac{100}{(s+1)^3}$ in rad/s is

(*a*) $\sqrt{3}$ (*b*) $\frac{1}{\sqrt{3}}$

(*c*) 3 (*d*) $3\sqrt{3}$

157. Consider the following asymptotic Bode magnitude plot (ω is in rad/s).



Which one of the following transfer functions is best represented by the above Bode magnitude plot?

(*a*) $\dfrac{2s}{(1+0.5s)(1+0.25s)^2}$ (*b*) $\dfrac{4(1+0.5s)}{s(1+0.25s)}$

(*c*) $\dfrac{2s}{(1+2s)(1+4s)}$ (*d*) $\dfrac{4s}{(1+2s)(1+4s)^2}$

158. Loop transfer function of a feedback system is $G(s)H(s) = \dfrac{s+3}{s^2(s-3)}$. Take the Nyquist contour in the clockwise direction. Then the Nyquist plot of $G(s)$ encircles $-1 + j0$

(*a*) once in clockwise direction

(*b*) twice in clockwise direction

(*c*) once in anticlockwise direction

(*d*) twice in anticlockwise direction

159. The value of the integral $2\int_{-\infty}^{\infty}\left(\dfrac{\sin 2\pi t}{\pi t}\right)dt$ is equal to

(*a*) 0 (*b*) 0.5

(*c*) 1 (*d*) 2

160. Let $P = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$. Consider the set S of all vectors $\begin{pmatrix} x \\ y \end{pmatrix}$ such that $a^2 + b^2 = 1$ where $\begin{pmatrix} a \\ b \end{pmatrix} = P\begin{pmatrix} x \\ y \end{pmatrix}$.

Then S is

(*a*) a circle of radius $\sqrt{10}$

(*b*) a circle of radius $\dfrac{1}{\sqrt{10}}$

(*c*) an ellipse with major axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

(*d*) an ellipse with minor axis along $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

161. The open loop transfer function of a unity feedback control system is given by

$$G(s) = \frac{K(s+1)}{s(1+Ts)(1+2s)}, K > 0, T > 0$$

The closed loop system will be stable if,

(*a*) $0 < T < \dfrac{4(K+1)}{K-1}$ (*b*) $0 < K < \dfrac{4(T+2)}{T-2}$

(*c*) $0 < K < \dfrac{T+2}{T-2}$ (*d*) $0 < T < \dfrac{8(K+1)}{K-1}$

162. A second-order real system has the following properties :

A. the damping ratio $\xi = 0.5$ and undamped natural frequency $\omega_n = 10$ rad/s,

B. the steady state value of the output, to a unit step input, is 1.02.

The transfer function of the system is

(*a*) $\dfrac{1.02}{s^2+5s+100}$ (*b*) $\dfrac{1.02}{s^2+10s+100}$

(*c*) $\dfrac{100}{s^2+10s+100}$ (*d*) $\dfrac{102}{s^2+5s+100}$

163. The gain at the breakaway point of the root locus of a unity feedback system with open loop transfer function $G(s) = \dfrac{Ks}{(s-1)(s-4)}$ is

(*a*) 1 (*b*) 2

(*c*) 5 (*d*) 9

2017

164. A closed loop system has the characteristic equation given by $s^3 + Ks^2 + (K+2)s + 3 = 0$. For this system to be stable, which one of the following conditions should be satisfied?

(*a*) $0 < K < 0.5$ (*b*) $0.5 < K < 1$

(*c*) $0 < K < 1$ (*d*) $K > 1$

165. The transfer function of a system is given by, $\dfrac{V_0(s)}{V_1(s)} = \dfrac{1-s}{1+s}$. Let the output of the system be $v_0(t) = V_m \sin(\omega t + \varphi)$ for the input, $v_i(t) = V_m \sin(\omega t)$. Then the minimum and maximum values of φ (in radians) are respectively

(*a*) $\dfrac{-\pi}{2}$ and $\dfrac{\pi}{2}$ (*b*) $\dfrac{-\pi}{2}$ and 0

(*c*) 0 and $\dfrac{\pi}{2}$ (*d*) $-\pi$ and 0

166. The transfer function of the system $Y(s)/U(s)$ whose state-space equations are given below is:

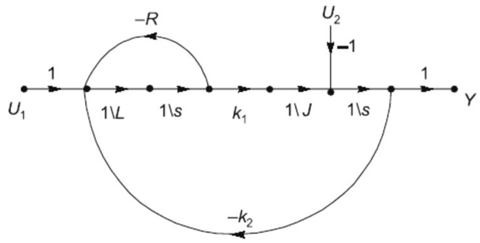
$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix}u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

(*a*) $\frac{(s+2)}{(s^2-2s-2)}$ (*b*) $\frac{(s-2)}{(s^2+s-4)}$

(*c*) $\frac{(s-4)}{(s^2+s-4)}$ (*d*) $\frac{(s+4)}{(s^2-s-4)}$

167. In the system whose signal flow graph is shown in figure, $U_1(s)$ and $U_2(s)$ are inputs. The transfer function $\frac{Y(s)}{U_1(s)}$ is



(*a*) $\frac{k_1}{JLs^2+JRs+k_1k_2}$

(*b*) $\frac{k_1}{JLs^2-JRs-k_1k_2}$

(*c*) $\frac{k_1-U_2(R+sL)}{JLs^2+(JR-U_2L)s+k_1k_2-U_2R}$

(*d*) $\frac{k_1-U_2(sL-R)}{JLs^2-(JR+U_2L)s-k_1k_2+U_2R}$

2018

168. Consider a lossy transmission line with V_1 and V_2 as the sending and receiving end voltages, respectively. Z and X are the series impedance and reactance of the line, respectively. The steady-state stability limit for the transmission line will be

(*a*) greater than $\left|\frac{V_1V_2}{X}\right|$ (*b*) less than $\left|\frac{V_1V_2}{X}\right|$

(*c*) equal to $\left|\frac{V_1V_2}{X}\right|$ (*d*) equal to $\left|\frac{V_1V_2}{Z}\right|$

169. Match the transfer functions of the second-order systems with the nature of the system given below.

Transfer functions	Nature of system
P. $\frac{15}{s^2+5s+15}$	I. Overdamped
Q. $\frac{25}{s^2+10s+25}$	II. Critically damped
R. $\frac{35}{s^2+18s+35}$	III. Underdamped

(*a*) P-I, Q-II, R-III (*b*) P-II, Q-I, R-III

(*c*) P-III, Q-II, R-I (*d*) P-III, Q-I, R-II

170. The number of roots of polynomial, $s^7+s^6+7s^5+14s^4+31s^3+73s^2+25s+200$, in the open left half of the complex plane is

(*a*) 3 (*b*) 4

(*c*) 5 (*d*) 6

2019

171. The characteristic equation of a linear time-invariant (LTI) system is given by

$$\Delta(s) = s^4+3s^3+3s^2+s+k=0$$

The system BIBO stable if

(a) $k > 3$ (b) $0 < k < \frac{8}{9}$

(c) $0 < k < \frac{12}{9}$ (d) $k > 6$

172. A system transfer function is

$$H(s) = \frac{a_1s^2+b_1s+c_1}{a_2s^2+b_2s+c_2}.$$

If $a_1 = b_1 = 0$, and all other coefficients are positive, the transfer function represents a

(a) high pass filter (b) notch filter

(c) low pass filter (d) band pass filter

173. The open loop transfer function of a unity feedback system is given by

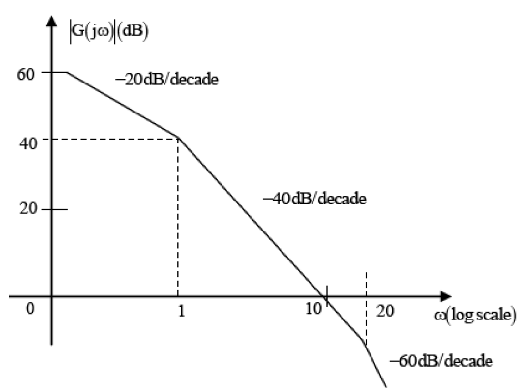
$$G(s) = \frac{\pi e^{-0.25s}}{s},$$

In G(s) plane, the Nyquist plot of G(s) passes through the negative real axis at the point.

(a) (–1.5, j0) (b) (–0.5, j0)

(c) (–0.75, j0) (d) (–1.25, j0)

174. The asymptotic Bode magnitude plot of a minimum phase transfer function G(s) is shown below.



Consider the following two statements.

Statement I : Transfer function G(s) has three poles and one zero.

Statement II : At very high frequency ($\omega \to \infty$), the phase angle $\angle G(j\omega) = -\frac{3\pi}{2}$

Which one of the following option is correct?

(*a*) Statement I is false and statement II is true.

(*b*) Both the statements are true.

(*c*) Both the statements are false.

(*d*) Statement I is true and statement II is false.

175. The transfer function of a phase lead compensator is given by $D(s) = \dfrac{3\left(s+\frac{1}{3T}\right)}{\left(s+\frac{1}{T}\right)}$

The frequency (in rad/sec), at which $\angle D(j\omega)$ is maximum, is

(*a*) $\sqrt{3T^2}$ (*b*) $\sqrt{\dfrac{3}{T^2}}$

(*c*) $\sqrt{3T}$ (*d*) $\sqrt{\dfrac{1}{3T^2}}$

176. Consider a state-variable model of a system

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}0 & 1\\ -\alpha & -2\beta\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \alpha\end{bmatrix}r\ ;\ y = [1\ \ 0]\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

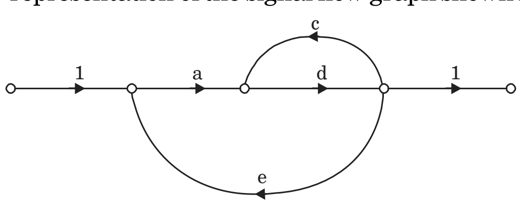
Where y is the output, and r is the input. The damping ratio ξ and the undamped natural frequency ω_n (rad/sec) of the system are given by

(a) $\xi = \sqrt{\alpha};\ \omega_n = \dfrac{\beta}{\sqrt{\alpha}}$ (b) $\xi = \dfrac{\sqrt{\alpha}}{\beta};\ \omega_n = \sqrt{\beta}$

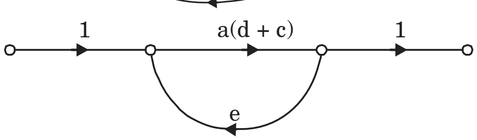
(c) $\xi = \sqrt{\beta};\ \omega_n = \sqrt{\alpha}$ (d) $\xi = \dfrac{\beta}{\sqrt{\alpha}};\ \omega_n = \sqrt{\alpha}$

2020

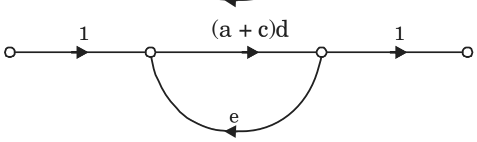
177. Which of the options is an equivalent representation of the signal flow graph shown here



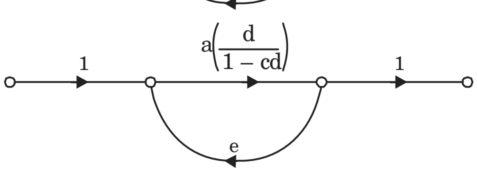
(a)



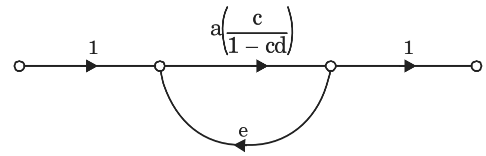
(b)



(c)



(d)

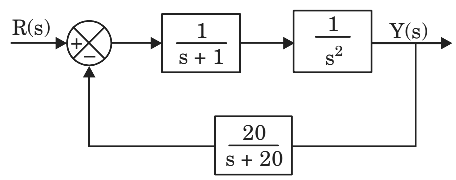


178. Consider a permanent magnet dc (PMDC) motor which is initially at rest. At t = 0, a dc voltage of 5 V is applied to the motor. Its speed monotonically increases from 0 rad/s to 6.32 rad/s in 0.5 s and finally settles to 10 rad/s. Assuming that the armature inductance of the motor is negligible, the transfer function of the motor is

(a) $\dfrac{10}{0.5s+1}$ (b) $\dfrac{2}{0.5s+1}$

(c) $\dfrac{2}{s+0.5}$ (d) $\dfrac{10}{s+0.5}$

179. Which of the following options is correct for the system shown below?



(a) 4^{th} order and stable

(b) 3^{rd} order and stable

(c) 4^{th} order and unstable

(d) 3^{rd} order and unstable

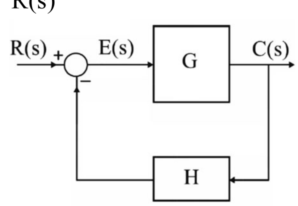
180. Consider a negative unity feedback system with the forward path transfer function $\dfrac{s^2+s+1}{s^3+2s^2+2s+K}$, where K is a positive real number. The value of K for which the system will have some of its poles in the imaginary axis is ____.

(a) 9 (b) 8

(c) 7 (d) 6

2021

181. For the closed-loop system shown, the transfer function $\dfrac{E(s)}{R(s)}$ is



(a) $\dfrac{G}{1+GH}$ (b) $\dfrac{GH}{1+GH}$

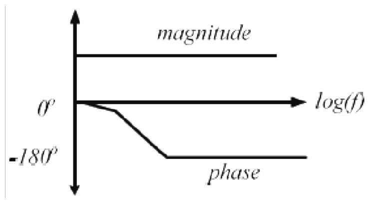
(c) $\dfrac{1}{1+GH}$ (d) $\dfrac{1}{1+G}$

2022

182. The transfer function of a real system, H(s), is given as : $H(s) = \frac{As + B}{s^2 + Cs + D}$, where A, B, C and D are positive constants. This system cannot operate as

(a) low pass filter. (b) high pass filter.
(c) band pass filter. (d) an integrator.

183. The Bode magnitude plot of a first order stable system is constant with frequency. The asymptotic value of the high frequency phase, for the system, is –180°. This system has



(a) one LHP pole and one RHP zero at the same frequency.
(b) one LHP pole and one LHP zero at the same frequency.
(c) two LHP poles and one RHP zero.
(d) two RHP poles and one LHP zero.

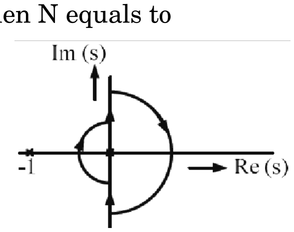
184. The open loop transfer function of a unity gain negative feedback system is given by $G(s) = \frac{k}{s^2 + 2s - 5}$. The range of k for which the system is stable, is

(a) $k > 3$ (b) $k < 3$
(c) $k > 5$ (d) $k < 5$

185. The open loop transfer function of a unity gain negative feedback system is given as

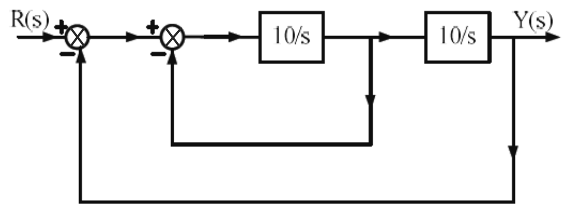
$$G(s) = \frac{1}{s(s+1)}$$

The Nyquist contour in the s-plane encloses the entire right half plane and a small neighbourhood around the origin in the left half plane, as shown in the figure below. The number of encirclements of the point (–1 + j0) by the Nyquist plot of G(s), corresponding to the Nyquist contour, is denoted as N. Then N equals to



(a) 0 (b) 1
(c) 2 (d) 3

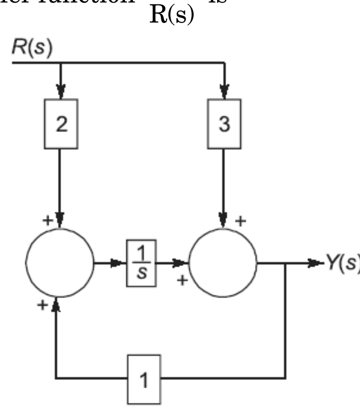
186. The damping ratio and undamped natural frequency of a closed loop system as shown in the figure, are denoted as ξ and ω_n, respectively. The values of ξ and ω_n are



(a) $\xi = 0.5$ and $\omega_n = 10$ rad/s
(b) $\xi = 0.1$ and $\omega_n = 10$ rad/s
(c) $\xi = 0.707$ and $\omega_n = 10$ rad/s
(d) $\xi = 0.707$ and $\omega_n = 100$ rad/s

2023

187. For the block diagram shown in the figure, the transfer function $\frac{Y(s)}{R(s)}$ is

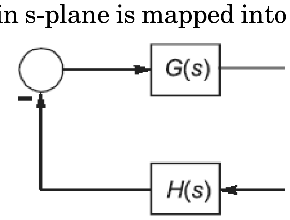


(a) $\frac{2s+3}{s+1}$ (b) $\frac{3s+2}{s-1}$
(c) $\frac{s+1}{3s+2}$ (d) $\frac{3s+2}{s+1}$

188. In the Nyquist plot of the open-loop transfer function

$$G(s)H(s) = \frac{3s+5}{s-1}$$

corresponding to the feedback loop shown in the figure, the infinite semi-circular arc of the Nyquist contour in s-plane is mapped into a point at



(a) $G(s)H(s) = \infty$ (b) $G(s)H(s) = 0$
(c) $G(s)H(s) = 3$ (d) $G(s)H(s) = -5$

189. Consider a unity-gain negative feedback system consisting of the plant G(s) (given below) and a proportional-integral controller. Let the proportional gain and integral gain be 3 and 1, respectively. For a unit step reference input, the final values of the controller output and the plant output, respectively, are

$$G(s) = \frac{1}{s-1}$$

(a) ∞, ∞ (b) 1, 0

(c) 1, –1 (d) –1, 1

190. Consider a lead compensator of the form

$$K(s) = \frac{1+\frac{s}{\alpha}}{1+\frac{s}{\beta\alpha}}, \beta > 1, \alpha > 0$$

The frequency at which this compensator produces maximum phase lead is 4 rad/s. At this frequency, the gain amplification provided by the controller, assuming asymptotic Bode-magnitude plot of K(s), is 6 dB. The values of α, β, respectively, are

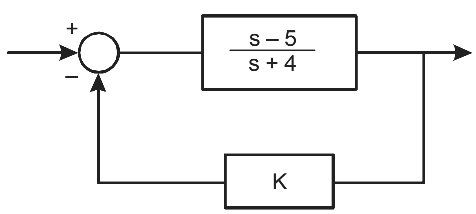
(a) 1, 16 (b) 2, 4

(c) 3, 5 (d) 2.66, 2.25

NUMERICAL TYPE QUESTIONS

1992

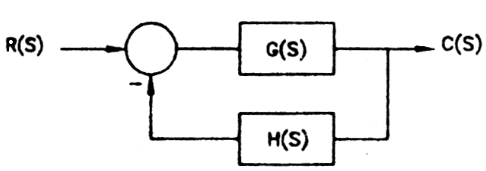
1. For what range of K is the following system (Figure) asymptotically stable ? Assume $K \geq 0$



1994

Direction (Q. 2) : Indicate whether the following statement is TRUE or FALSE. Write the indicating work fully and legibly. A 'FALSE' answer must be accompanied by a very brief (preferably one or two sentences) justification.

2. The closed loop system, of Figure, is stable if the transfer function $T(s) = \frac{C(s)}{R(s)}$ is stable.



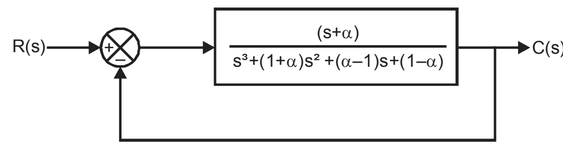
3. The number of positive real roots of the equation $s^3 - 2s + 2 = 0$ is ________

1995

4. Closed loop stability implies that [1 + G(s) H(s)] has only _________ in the left half of the s-plane.

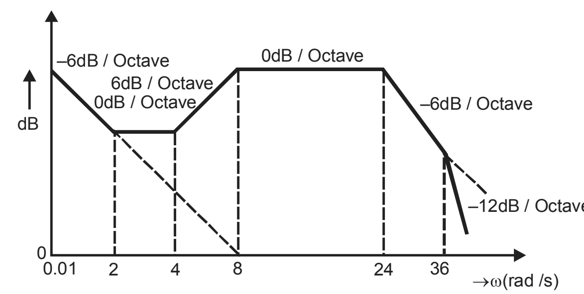
2014

5. For the given system, it is desired that the system be stable. The minimum value of α for this condition is ________.



6. The Bode magnitude plot of the transfer function $G(s) = \frac{K(1+0.5s)(1+as)}{s\left(1+\frac{s}{8}\right)(1+bs)\left(1+\frac{s}{36}\right)}$ I shown below:

Note that – 6 dB/octave = – 20dB/decade. The value of $\frac{a}{bK}$ is ____________



7. The closed-loop transfer function of a system is $T(S) = \frac{4}{(s^2+0.4s+4)}$. The steady state error due to unit step input is ____.

8. A system with the open loop transfer function

$$G(s) = \frac{K}{s(s+2)(s^2+2s+2)}$$

is connected in a negative feedback configuration with a feedback gain of unity. For the closed loop system to be marginally stable, the value of K is ____

2015

9. An open loop control system results in a response of $e^{-2t}(\sin 5t + \cos 5t)$ for a unit impulse input. The DC gain of the control system is ______.

2016

10. Consider a linear time-invariant system with transfer function

$$H(s) = \frac{1}{(s+1)}$$

If the input is cos (t) and the steady state output is A cos $(t + \alpha)$, then the value of A is _____.

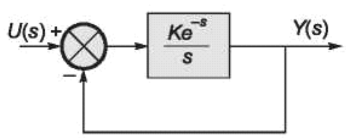
11. The line integral of the vector field

$$F = 5xz\hat{i} + \left(3x^2 + 2y\right)j + x^2zk$$

along a path from (0, 0, 0) to (1, 1, 1) parameterized by (t, t^2, t) is _________.

2017

12. Consider the unity feedback control system shown. The value of K that results in a phase margin of the system to be 30° is _______. (Give the answer up to two decimal places.)



13. For a system having transfer function $G(s) = \frac{-s+1}{s+1}$, a unit step input is applied at time $t = 0$. The value of the response of the system at $t = 1.5$ sec (rounded off to three decimal places) is______.

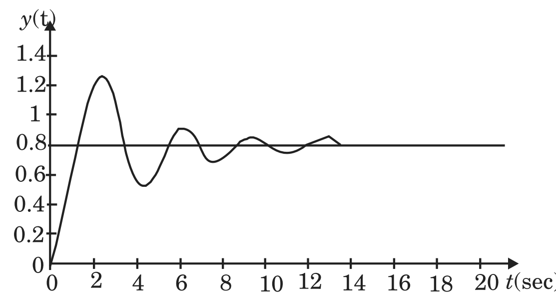
2018

14. Consider a unity feedback system with forward transfer function given by

$$G(s) = \frac{1}{(s+1)(s+2)}$$

The steady-state error in the output of the system for a unit-step input is_____(up to 2 decimal places).

15. The unit step response y(t) of a unity feedback system with open-loop transfer function G(s)H(s) $= \frac{K}{(s+1)^2(s+2)}$ is shown in the figure. The value of K is _________ (upto 2 decimal places).

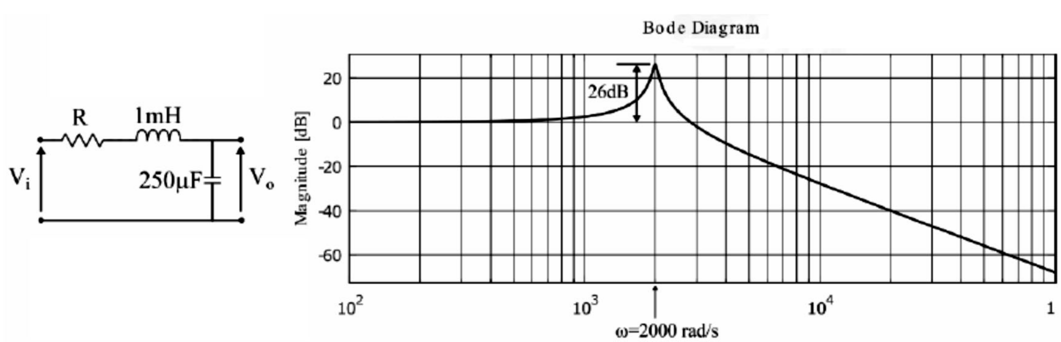


2020

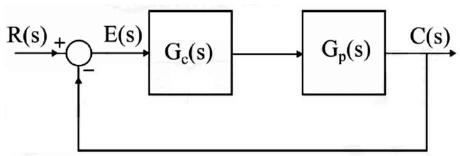
16. Consider a negative unity feedback system with forward path transfer function $G(s) = \frac{K}{(s+a)(s-b)(s+c)}$, where K, a, b, c are positive real numbers. For a Nyquist path enclosing the entire imaginary axis and right half of the s-plane in the clockwise direction, the Nyquist plot of [1 + G(s)], encircles the origin of [1 + G(s)]-plane once in the clockwise direction and never passes through this origin for a certain value of 'K'. Then the number of poles of $\frac{G(s)}{1+G(s)}$ lying in the open right half of the s-plane is _____.

2021

17. The Bode magnitude plot for the transfer function $\frac{V_o(s)}{V_i(s)}$ of the circuit is as shown. The value of R is _____ Ω. (Round off to 2 decimal places.)



18. Consider a closed-loop system as shown, $G_p(s) = \dfrac{14.4}{s(1+0.1s)}$ is the plant transfer function and $G_c(s) = 1$ is the compensator. For a unit-step input, the output response has damped oscillations. The damped natural frequency is ________ rad/s. (Round off to 2 decimal places).



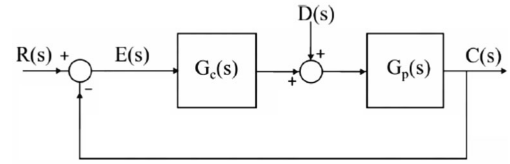
19. In the given figure, plant

$$G_p(S) = \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)} \text{ and compensator}$$

$$G_c(S) = K\left(\frac{1+T_1 s}{1+T_2 s}\right).$$

The external disturbance input is D(s). It is desired that when the disturbance is a unit step, the steady-state error should not exceed 0.1 unit. The minimum value of K is ________.

(Round off to 2 decimal places.)



20. The state space representation of a first-order system is given as

$\dot{x} = -x + u$

$y = x$

where, x is the state variable, u is the control input and y is the controlled output. Let $u = -Kx$ be the control law, where K is the controller gain. To place a closed-loop pole at –2, the value of K is ______.

ANSWERS

MCQ Type Questions

1. (b)	**2.** (b)	**3.** (a)	**4.** (a)	**5.** (a)	**6.** (c)	**7.** (a)	**8.** (*)	**9.** (a)	**10.** (a)
11. (b)	**12.** (c)	**13.** (c)	**14.** (d)	**15.** (c)	**16.** (b)	**17.** (b)	**18.** (d)	**19.** (b)	**20.** (d)
21. (b)	**22.** (c)	**23.** (a)	**24.** (a)	**25.** (c)	**26.** (a)	**27.** (d)	**28.** (c)	**29.** (a)	**30.** (c)
31. (c)	**32.** (b)	**33.** (d)	**34.** (a)	**35.** (d)	**36.** (c)	**37.** (a)	**38.** (c)	**39.** (d)	**40.** (a)
41. (c)	**42.** (a)	**43.** (b)	**44.** (c)	**45.** (c)	**46.** (c)	**47.** (b)	**48.** (b)	**49.** (a)	**50.** (b)
51. (d)	**52.** (c)	**53.** (a)	**54.** (a)	**55.** (b)	**56.** (a)	**57.** (d)	**58.** (a)	**59.** (c)	**60.** (a)
61. (b)	**62.** (a)	**63.** (d)	**64.** (c)	**65.** (c)	**66.** (b)	**67.** (a)	**68.** (d)	**69.** (d)	**70.** (b)
71. (c)	**72.** (c)	**73.** (a)	**74.** (d)	**75.** (c)	**76.** (c)	**77.** (b)	**78.** (c)	**79.** (d)	**80.** (b)
81. (a)	**82.** (c)	**83.** (b)	**84.** (d)	**85.** (a)	**86.** (b)	**87.** (c)	**88.** (a)	**89.** (d)	**90.** (b)
91. (d)	**92.** (b)	**93.** (d)	**94.** (d)	**95.** (a)	**96.** (a)	**97.** (d)	**98.** (b)	**99.** (a)	**100.** (c)
101. (c)	**102.** (c)	**103.** (d)	**104.** (d)	**105.** (d)	**106.** (b)	**107.** (d)	**108.** (d)	**109.** (a)	**110.** (c)
111. (d)	**112.** (a)	**113.** (c)	**114.** (c)	**115.** (a)	**116.** (a)	**117.** (d)	**118.** (b)	**119.** (b)	**120.** (a)
121. (d)	**122.** (a)	**123.** (c)	**124.** (d)	**125.** (d)	**126.** (a)	**127.** (b)	**128.** (d)	**129.** (b)	**130.** (c)
131. (c)	**132.** (c)	**133.** (c)	**134.** (b)	**135.** (a)	**136.** (b)	**137.** (b)	**138.** (c)	**139.** (c)	**140.** (a)
141. (c)	**142.** (c)	**143.** (a)	**144.** (b)	**145.** (a)	**146.** (c)	**147.** (b)	**148.** (d)	**149.** (b)	**150.** (c)
151. (d)	**152.** (d)	**153.** (b)	**154.** (a)	**155.** (b)	**156.** (a)	**157.** (a)	**158.** (a)	**159.** (d)	**160.** (c)
161. (c)	**162.** (b)	**163.** (a)	**164.** (d)	**165.** (d)	**166.** (d)	**167.** (a)	**168.** (b)	**169.** (c)	**170.** (a)
171. (b)	**172.** (c)	**173.** (b)	**174.** (a)	**175.** (d)	**176.** (d)	**177.** (c)	**178.** (b)	**179.** (c)	**180.** (b)
181. (c)	**182.** (b)	**183.** (a)	**184.** (c)	**185.** (b)	**186.** (a)	**187.** (b)	**188.** (c)	**189.** (d)	**190.** (b)

Numerical Type Questions

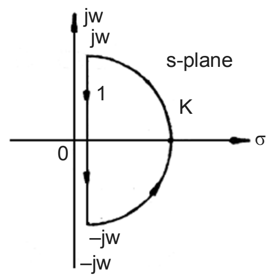
1. ($K \le 4/5$)	**2.** (True)	**3.** (One)	**4.** (roots)	**5.** (0.618)	**6.** (0.75)
7. (0 to 0)	**8.** (5to5)	**9.** 0.241	**10.** (0.707)	**11.** (4.41)	**12.** (1.047)
13. (0.554)	**14.** 0.66	**15.** 8	**16.** (2 to 2)	**17.** (0.09 to 0.11)	
18. (10.80 to11.00)		**19.** (9.50 to 9.60)	**20.** (1 to 1)		

EXPLANATIONS

MCQ TYPE QUESTIONS

1. Given : $G(s) = \dfrac{1}{(s-1)(s+2)(s+3)}$

Since the open-loop transfer function has no poles at the origin, the Nyquist path is as shown below :



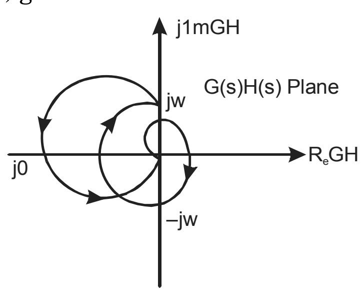
Path I : $G(j\omega) = \dfrac{1}{(j\omega-1)(j\omega+2)(j\omega+3)}$

$$G(j0) = -\frac{1}{6} = \frac{1}{6}\angle 189^\circ$$

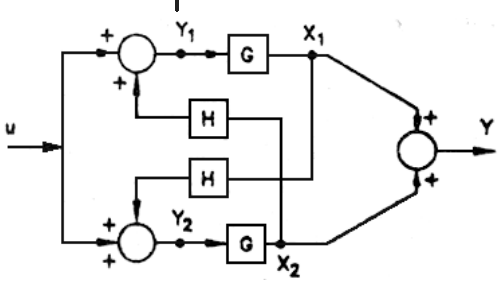
Path II : $G(j\omega) = \dfrac{1}{(j\omega)^3} = \dfrac{1}{(\mathrm{Re}^{j0})^3} \quad R \to \infty$

$$= \delta_e^{-j30} \qquad \delta \to 0$$

Hence as ϕ goes from – 90° to + 90° in path II, $G(j\omega)$ goes from 270° to – 270° as shown below



2.



$$Y_1 = U + X_2H$$
$$Y_2 = U + X_1H$$

and $X_1 = GY_1 = GU + GX_2H$

$$X_2 = GY_2 = GU + GX_1H$$

and $Y = X_1 + X_2$

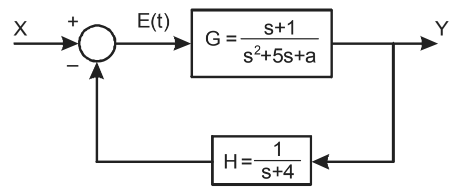
$$= 2GU + GH(X_1 + X_2)$$
$$= 2GU + GHY$$

or $Y = \dfrac{2GU}{1-GH}$

or $\dfrac{Y}{U} = \dfrac{2G}{1-GH}$

3. Only (*a*) is a valid root loci since in (*b*), (*c*) and (*d*) the root loci lies to the left of an even number of zeros and poles.

4.
$$E_{(s)} = X_{(s)} - Y_{(s)}H$$
$$= X_{(s)} - \frac{G}{1+GH}H.X_{(s)}$$
$$= X_{(s)}\frac{1}{1+GH}$$
$$X_{(s)} = \frac{1}{s}$$



Hence $E_{(s)} = \dfrac{1}{s(1+GH)}$

$$= \frac{1}{s}\left[\frac{1}{1+\dfrac{s+1}{(s+4)(s^2+5s+a)}}\right]$$

The steady state error,

$$e_{ss} = \lim_{t\to\infty} \in_{(t)}$$
$$= \lim_{t\to\infty} sE_{(s)}$$
$$= \lim_{t\to\infty} \frac{1}{1+\dfrac{s+1}{(s+4)(s^2+5s+a)}}$$
$$= \frac{1}{1+\dfrac{1}{4a}} = \frac{4a}{4a+1} = 0$$

or $a = 0$

5. (*a*) $H(s) = \dfrac{s}{s+1} = \dfrac{s+1-1}{s+1} = 1 - \dfrac{1}{s+1}$

$$h(t) = \delta_{(t)} - e^{-t}u_{(t)}$$

This matches best with (P)

(*b*) $H(s) = \dfrac{s}{(s+1)^2}$

$$h(t) = t\,e^{-t}u_{(t)}$$

This matches best with (Q)

(*c*) $\quad H(s) = \dfrac{1}{s^2+s+1} = \dfrac{\omega_n^2}{s^2+2\delta\omega_n s+w_n^2}$

Comparing (1) and (2), we get

$$\omega_n = 1,$$

$$2\delta\omega_n = 1,$$

$$\Rightarrow \quad \delta = \frac{1}{2}$$

$$h(t) = \frac{\omega_n}{\sqrt{1-\delta^2}} e^{-\delta\,\omega_{n.}t} \sin(\omega_n\sqrt{1-\delta^2}t)$$

$$= \frac{1}{\sqrt{\frac{3}{4}}} e^{-1/2} \sin\left(\sqrt{\frac{3}{4}}t\right)$$

This matches best with (*S*)

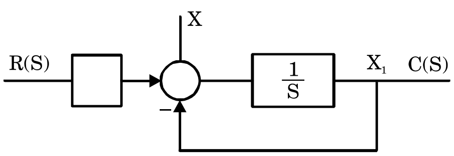
(*d*) $\quad H(s) = \dfrac{1}{s^2+1}$

$$h(t) = \sin(t)$$

Such matches best with (R)

Hence, $a - P$, $\quad b - Q$, $\quad c - S$, $\quad d - R$

6.



From the figure, $x_1 = -x_1 + 3r(t)$

The A matrix is (–1) and hence choice (*c*) is correct.

7. The transfer function of a phase–lead compensator is

$$\frac{E_2(s)}{E_1(s)} = \frac{1}{a}\frac{(1+aTs)}{1+Ts}, \quad a > 1$$

$$\text{Zero is at } s = -\frac{1}{aT};$$

$$\textbf{\textit{Pole}} \text{ is at } s = -\frac{1}{T}$$

Hence choice (*a*) is correct, since $a > 1$.

8. $a - R$, $\quad b - S, c - Q$, $\quad d - P$

9.
$$\frac{C(s)}{R(s)} = \frac{2(s-1)}{(s+2)(s+1)};$$

$$R(s) = \frac{1}{s}$$

Hence
$$C(s) = \frac{2(s-1)}{(s+1)(s+2)}$$

$$= \frac{k_1}{s} + \frac{k_2}{s+1} + \frac{k_3}{s+3},$$

expanding in partial fractions

$$k_1 = \frac{2(s-1)}{(s+1)(s+2)}\Big|_{s=0} = \frac{-2}{2} = -1$$

$$k_2 = \frac{2(s-1)}{s(s+2)}\Big|_{s=-1} = \frac{2(-2)}{-1(1)} = 4$$

$$k_3 = \frac{2(s-1)}{s(s+2)}\Big|_{s=-2} = \frac{2(-3)}{-2(-1)} = -3$$

Hence $C(s) = \dfrac{1}{s} + \dfrac{4}{s+1} - \dfrac{3}{s+2}$

and the output $c(t) = [-1 + 4e^{-t} - 3e^{-2t}]u(t)$.

10. The transfer function G(*s*) is :

$$G(s) = C(sI - A)^{-1}B$$

$$(sI - A)\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -4 & -1 \\ 3 & -1 \end{bmatrix} = \begin{bmatrix} s+4 & 1 \\ -3 & s+1 \end{bmatrix}$$

$$(sI - A) = \frac{\begin{bmatrix} s+1 & 3 \\ -1 & s+4 \end{bmatrix}}{s^2+5s+4+3} = \frac{\begin{bmatrix} s+1 & -1 \\ 3 & s+4 \end{bmatrix}}{s^2+5s+7}$$

$$\text{Hence } G(s)\ \frac{[1\ 0]\begin{bmatrix} s+1 & 3 \\ -1 & s+4 \end{bmatrix}}{s^2+5s+7}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{[s+1-1]}{s^2+5s+7}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{s}{s^2+5s+7}$$

11. $\quad C(t) = -te^{-t} + 2e^{-t},$

$(t \geq 0)$

$$C(s) = -\frac{1}{(s+1)^2} + \frac{2}{s+1^2} = \frac{2s+1}{(s+1)^2}$$

$$C(s) = \frac{G(s)}{1+G(s)},$$

$$G(s) = \frac{C(s)}{1-C(s)} = \frac{\frac{2s+1}{(s+1)^2}}{1-\frac{2s+1}{(s+1)^2}} = \frac{2s+1}{s^2}$$

12.
$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)},$$

$$\frac{G(s)}{1-G(s)} = \frac{\frac{1}{s(s+1)}}{1+\frac{1}{s(s+1)}} = \frac{1}{s^2+s+1}$$

$$\omega_n = 1, \qquad \omega_n = \frac{1}{2}, \qquad \zeta = \frac{1}{2}$$

$$M_p = e^{-\xi\sqrt{1-\zeta^2}} = e^{(-\pi/2)/\sqrt{1-(1/4)}}$$

$$= e^{-\pi/\sqrt{3}} = 0.163$$

14.
$$\frac{C(s)}{R(s)} = \frac{1}{1+s}, \qquad \frac{C(j\omega)}{R(j\omega)} = \frac{1}{1+j\omega}$$

From $r(t)\sin t$, $\omega = 1$, $\left|\dfrac{C(j\omega)}{R(j\omega)}\right| = \dfrac{1}{\sqrt{1+1}} = \dfrac{1}{\sqrt{2}}$

$$\angle\frac{C(j\omega)}{R(j\omega)} = -\tan^{-1}1 = -\frac{\pi}{4}$$

$$\therefore \text{ SS value of } c(t) = \frac{1}{\sqrt{2}}\sin\left(t - \frac{\pi}{4}\right)$$

15. $$\frac{C(s)}{R(s)} = \frac{\frac{K}{s(s+2)}}{1+\frac{K}{s(s+2)}(1+as)}$$

$$= \frac{K}{s(s+2)+K(1+as)}$$

$$= \frac{K}{s^2+s(2+Ka)+K}$$

$$K = \omega_n^2 = 16$$

$$2\zeta\omega_n = 2 + Ka = 2 \times 0.7 \times 4 = 5.6$$

$$\therefore \quad a = \frac{5.6-2}{16} = \mathbf{0.225}$$

16. Step response is integral of unit impulse response

$$u(t) = \int_o^t (-4e^{-t} + 6e^{-2t})\,dt$$

$$= [4e^{-t} - 3e^{-2t}]^t_0$$

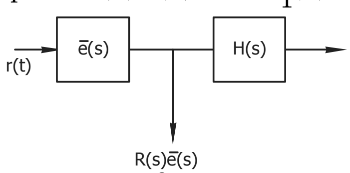
$$= 4e^{-t} - 3e^{-2t} - 1.$$

18. Linear time invariant control system is shown as below.

X (t)	Y (t)
r (t)	C (t)

We know, $C(S) = H(S)\,R(S)$

$\therefore$ Output is $H(S)\,R(S)\,e^{-S} = C_1(S)$



or $\quad C_1(S) = e^{-S}\,C(S)$

$\Rightarrow \quad C_1(t) = C(t)\,u\,(t-1)$

19. For a linear control system with no poles in R. H. S. of S-plane including roots on $j\omega$ axis with bounded input, the output may be unbounded.

20. System becomes fast and hence t_r is less

21. Solution exists $Ax = b$, b should be in column space of A.

22. $$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}; \quad X_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

From $\quad (A - \lambda I)\,X_1 = 0$

$$\begin{bmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & -\lambda \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = 0$$

or $$\begin{bmatrix} -2-\lambda+4+3 \\ 2+2\lambda+6 \\ -1-4+\lambda \end{bmatrix} = 0$$

or $\quad \lambda = \mathbf{5}$

23. Sum of eigen values of A

= sum of diagonal element

$= 2 + 1 + 3 + 4 = 10$

24. Let output of summer be K (S)

$$K(S) = \frac{C(S)}{G_2G_3}$$

$$\therefore \quad \frac{C(S)}{G_2G_3} = G_1\left[R(S) - \frac{C(S)H_1}{G_3}\right] - C\,(S)\,H_2$$

$$\therefore \quad C\,(S)\,[1 + H_1\,G_1\,G_2 + H_2\,G_2\,G_3] = G_1\,G_2\,G_3\,R\,(S).$$

25. Routh Hurwitz interion

S^4	2	3	7
S^3	1	5	
S^2	–7	7	
S^1	6		
S^0	7		

Since sign changes twice, therefore 2 roots in RHP.

26. det $\quad A = |A| = 5\,[3-0] - 0\,[0-2] + 2\,[0-6]$

$= 15 - 12 = 3$

$$\text{Adj}\,A = -\begin{vmatrix} \begin{vmatrix} 3 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & 2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 2 \\ 3 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & 0 \\ 2 & 1 \end{vmatrix} & \begin{vmatrix} 5 & 2 \\ 2 & 1 \end{vmatrix} & -\begin{vmatrix} 5 & 2 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & 3 \\ 2 & 0 \end{vmatrix} & -\begin{vmatrix} 5 & 0 \\ 2 & 0 \end{vmatrix} & \begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} \end{vmatrix}$$

$$\text{Adj}\,A = \begin{bmatrix} 3 & 0 & -6 \\ 0 & 1 & 0 \\ -6 & 0 & 15 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{\text{Adj}A}{|A|} = \frac{1}{3}\begin{bmatrix} 3 & 0 & -6 \\ 0 & 1 & 0 \\ -6 & 0 & 15 \end{bmatrix}$$

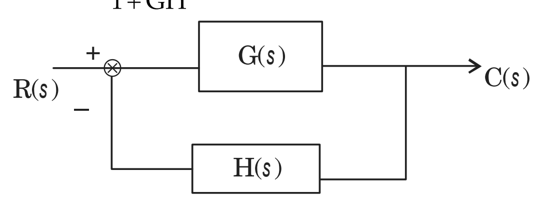
$$= \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1/3 & 0 \\ -2 & 0 & 5 \end{bmatrix}$$

28. $$S^T_G = -\frac{GH}{1+GH}, \; GH >> 1$$

$= -1$ (Sensitivity with change in H)

$$SG^T = \frac{1}{1+GH} \text{ (Sensitivity with change in G)}$$

29.



$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

For $H(s) = 1$ [unity feedback system]

$$E(s) = \frac{R(s) - C(s)}{R(s)} = \frac{1}{1+G(s)}$$

steady-state error, $G_{ss} = \underset{s\to 0}{lt}\ sE(s)$

$$= \underset{s\to 0}{lt}\ \frac{SR(s)}{1+G(s)}$$

For, $G(s) = \frac{1\,(s+Z_1)(s+Z_2)}{s\,(s+P_1)(s+P_2)}$ [Type -1 system]

and $R(s) = \frac{1}{s}$ [step function]

$$G_s = \underset{s\to 0}{lt}\ \frac{s}{1+k} = 0 \text{ [}k\text{ is a constant]}$$

30. $y(t) = te^{-t}$

$$y(s) = \pounds\left[y(t)\right] = +\frac{1}{(s+1)^2}$$

$$T(s) = \frac{Y(s)}{R(s)} = \frac{s}{(s+1)^2}$$

$[R(s) = \frac{1}{s}$, unit- step function].

31. Constructing routh- array, we get

5^4	2	3	10
5^3	1	5	0
5^2	−7	10	
5^1	$\frac{45}{7}$	$\frac{10}{7}$	
5^0	$\frac{520}{45}$	0	

As, there are no. of sign change is the first row is 2, therefore no. of roots in Rh-s plane is two.

32. Damping ratio =0.6, % overshoot

$$M_P = \left[e^{-\pi r}/\sqrt{1-r^2}\right] \times 100 = 10\%$$

33. Phase-angles $\phi = \angle D(j\omega) = \tan^{-1}(0.5\omega) - \tan^{-1}(0.05\,\omega)$

ϕ will be maximum when

$$\frac{d\phi}{d\omega} = 0 = \frac{0.5}{1+(0.5\omega)^2} - \frac{0.05}{1+(0.05\omega)^2}$$

$\Rightarrow$ $10 + 10\,(0.05\,\omega)^2 = 1 + (0.5\omega)^2$

$\Rightarrow$ $\omega^2 = \sqrt{\frac{9}{0.25 - 0.025}}$

$= \sqrt{40}$

$\omega = 6.32$ rad/sec.

and, $\phi = \tan^{-1}(6.32 \times 0.5) - \tan^{-1}(0.05 \times 6.32)$

$= 55°$

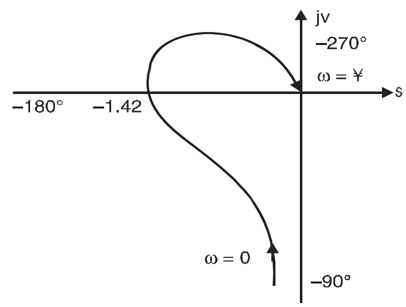
34. Phase margin :

$$\phi = 180° - \tan^{-1}\omega - \tan^{-1} 0.5\omega - 90°$$
$$= 180° - \tan^{-1}(0.466) - \tan^{-1}(0.233) - 90°$$
$$= 51.9°$$

35. For $\xi = 0.3$, value of K comes out a complex number, so it doesn't exist.

36. $a = +1.42$

GM (gain margin) = $-20 \log a$



Since $a > 1$,

So GM will be – ve and system unstable,

Now, $G(j\omega)\,H(j\omega) = \frac{1 - j\omega T_1}{j\omega\,(1 + j\omega\,T_2)}$

[assuming one pole in RHS plane].

$\angle G(j\omega)\,H(j\omega) = -90° - \tan\omega T - \tan\omega\,T_2$

At $\omega = 0$, $\angle G(j\omega)\,H(j\omega) = -90°$

At $\omega = \infty$, $\angle G(j\omega)\,H(j\omega) = -270°$

both the conditions satisfies by polor plot given.

37. $$\dot{X} = \begin{bmatrix} -3 & 1 \\ 0 & -2 \end{bmatrix} X = AX$$

Its solution is,

$$X(t) = \left[\pounds^{-1}\,[sI - A]^{-1}\right] X(0)$$

$$= \pounds^{-1}\begin{bmatrix} s+3 & -1 \\ 0 & s+2 \end{bmatrix}^{-1} \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

$$= \left[\pounds^{-1}\frac{\begin{bmatrix} s+2 & 1 \\ 0 & s+3 \end{bmatrix}}{(s+2)\,(s+3)}\right] X(0)$$

$$= \begin{bmatrix} \pounds^{-1}\frac{1}{s+3} & \pounds^{-1}\left[\frac{1}{s+2}\cdot\frac{1}{s+3}\right] \\ 0 & \pounds^{-1}\left[\frac{1}{s+2}\right] \end{bmatrix} X(0)$$

$$= \begin{bmatrix} e^{-3t} & e^{-2t} - e^{-3t} \\ 0 & e^{-2t} \end{bmatrix} \begin{bmatrix} 10 \\ -10 \end{bmatrix}$$

Nor, $X_{ss} = \lim_{t\to\infty} X(t)$

$$= \lim_{t\to\infty} \begin{bmatrix} 20e^{-3t} - 10e^{-2t} \\ -10e^{-2t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

38. Inductance of the circuit/km,

$$L = 4 \times 10^{-7} ln \frac{D}{r'} \; mH/km$$

$$= 4 \times 10^{-7} ln \frac{1}{0.7788 \times 5 \times 10^{-3}}$$

$$= 2.381 \; mH/km.$$

$\therefore$ Inductance $= 2.381 \; mH/km \times 10$

$= \mathbf{23.81 \; mH}$

39. Since at $\omega = 2$ and $\omega = 25$, slope changes from 40 dB/dec to –60 dB/dec at both the valve. So there are poles at $\omega = 2$ and $\omega = 25$. Also at $\omega = 0.1$ slope is – 40 dB/dec. It means, there is two poles at origin. Hence the transfer function, should be of the form,

$$T(s) = \frac{K(s+5)}{s^2(s+2)(s+25)}$$

Now, $54 = 20 \log \frac{5K}{(0.1)^2 \times 50}$

or $K = 50.$

Thus $T(s) = \frac{50\,(s+5)}{s^2\,(s+2)\,(s+25)}$

Common Data Q. (40 – 42)

40. Given, $G(s) = \frac{10{,}000}{s\,(s+10)^2}$

$H(s) = 1$

$$G\,(s)\,H\,(s) = \frac{10{,}000}{S\,(S+10)^2}$$

$$G\,(j\omega)\,H\,(j\omega) = \frac{10{,}000}{j\omega\,(j\omega+10)^2}$$

(*a*) G $(j\omega)$ in decibels

$$= 20 \log |\,G\,(j\omega)\,H\,(j\omega)\,|_{\omega=20}$$

$$= 20 \log \frac{10{,}000}{\{|20j||10+20j|^2\}}$$

$$= 20 \log \frac{10{,}000}{10000} = \mathbf{0 \; db.}$$

41. First, gain cross over frequency, $\omega = \omega_1$ should be calculated.

By definition,

As, $|G(j\omega)\,H\,(j\omega)|_{\omega=\omega_1} = 1$

$\Rightarrow \frac{10{,}000}{|j\,\omega_1\,|10+j\omega_1|^2} = 1$

$\Rightarrow 10{,}000 = \omega_1\,(\omega_1^{\;2} + 100)$

Gives $\omega_1 = 20$ rad/sec.

Now, phase margin,

$$\phi = \angle G\,(j\omega)\,H\,(j\omega)\,|_{\omega=\omega_1} + 180°$$

$$= -90° - 2\tan^{-1}\frac{\omega_1}{10} + 180° = \mathbf{-36.86°}$$

42. For, gain margin, first calculate phase-cross over frequency, $\omega = \omega_2$.

As per definition, $\angle G\,(j\omega)\,H\,(j\omega)\,|_{\omega=\omega_2} = -180°$

$\Rightarrow -90° - 2\,.\tan^{-1}\frac{\omega_2}{10} = -180°$

$\Rightarrow \tan^{-1}\frac{\omega_2}{10} = 45°$

or $\omega_2 = 10$ rad/sec.

Now, Gain margin, $GM = -20 \log a$

where $a = |\,G(j\omega)\,H\,(j\omega)\,|_{\omega=\omega_2}$

$$\therefore \quad GM = -20 \log \frac{10{,}000}{\omega_2\,(\omega_2^2 + 100)}$$

$$= -20 \log \frac{10{,}000}{10{,}200}$$

$$= \mathbf{-13.97 \; dB}$$

47. $\dot{X} = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} X + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$

$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad A = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

$AB = \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$\Rightarrow [B : AB] = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix}$

This is not a 2 × 2 matrix, hence system is uncontrollable

Now $[sI - A] = \begin{bmatrix} s & 0 \\ 5 & s \end{bmatrix} - \begin{bmatrix} 2 & 3 \\ 0 & 5 \end{bmatrix}$

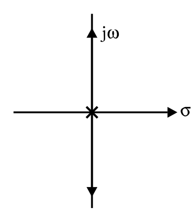
$= \begin{bmatrix} s-2 & -3 \\ 0 & s-5 \end{bmatrix} = (s-2)\,(s-5)$

Two roots on positive half of the y-axis, hence unstable

48. $G(s) = \frac{K}{s^2}$

Angle of asymptote, $\phi_A = \frac{\pm(2q+1)180}{2-0} = \pm 90$

Hence root locus can be



50. $\dot{X} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix} X + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u\,; \quad Y = [4\;0]\,X$

$[sI - A] = \begin{bmatrix} s-2 & 0 \\ 0 & s-4 \end{bmatrix}$

$[sI - A]^{-1} = \phi(s) \begin{bmatrix} \frac{1}{s-2} & 0 \\ 0 & \frac{1}{s-4} \end{bmatrix}$

$Y(s) = \phi\,(s)\,C\,(s)\,.\,X\,(s)$

$$\frac{Y(s)}{X(s)} = \begin{bmatrix} \frac{1}{s-2} & 0 \\ 0 & \frac{1}{s-4} \end{bmatrix} [4, 0] = \frac{4}{s-2}$$

With input as $\delta(t)$, $L[\delta(t)] = 1$

$$Y(s) = \frac{4}{s-2} \cdot [1] = \frac{4}{s-2}$$

$$\Rightarrow \quad Y(t) = 4e^{2t}$$

52. Taking Laplace-transform of both-sides, we have

$$[s^2 + 6s \times 5]\,X(s) = 12\left(\frac{1}{s} - \frac{1}{s+2}\right) = \frac{24}{s(s+2)}$$

or
$$X(s) = \frac{24}{s(s^2+6s+5)(s+2)}$$

then,
$$\lim_{t\to\infty} x(t) = \lim_{s\to 0} sX(s)$$

$$= \lim_{s\to 0} \frac{24}{(s^2+6s+5)(s+2)} = \mathbf{2.4}$$

53.
$$\theta = \tan^{-1}\frac{\omega}{a} - \tan^{-1}\frac{\omega}{b}$$

For lead compensator, $\theta > 0$

then, $b > a$

54. The state of the system at time t is,

$$X(t) = [sI - A]^{-1}X(0) = \phi(t)X(0)$$

$$= \begin{bmatrix} e^{-2t} & 0 \\ 0 & e^{-t} \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 2e^{-2t} \\ 3e^{-t} \end{bmatrix}$$

At $t = 1$, $X(1) = \begin{bmatrix} 2e^{-2} \\ 3e^{-1} \end{bmatrix} = \begin{bmatrix} 0.271 \\ 1.100 \end{bmatrix}$

55.
$$T(s) = \frac{C(s)}{R(s)} = \frac{1}{s^2 + \frac{s}{2} + \frac{1}{18}}$$

$$= \frac{18}{10s^2 + (s+1)} = \frac{18}{(3s+1)(6s+1)}$$

As, T(S) have the form, $\frac{A}{(J_1 s+1)(J_2 s+1)}$

The time - constant are $J_1 = 3$ sec, $J_2 = 6$ sec.

56.
$$\frac{Y(s)}{U(s)} = \frac{45}{s^2+16s+60}$$

Steady-state error, $e_{ss} = \lim_{s\to 0} sE(s)$

$$= \lim_{s\to 0} s\,[U(s) - Y(s)]$$

$$= \lim_{s\to 0} \frac{s^2+16s+15}{s^2+16s+60}$$

$$= \frac{1}{4} = 25\%.$$

57. The characteristic equation is,

$$1 + GH = 0$$

or
$$1 + \frac{45}{(s+15)(s+1)} = 0$$

or $s^2 + 16s + 60 = 0$

or $(s+6)(s+10) = 0$

Thus, $s = -6$, $s = -10$ are the roots.

58.
$$\frac{d^2\omega}{dt^2} = \frac{B}{J}\frac{d\omega}{dt} - \frac{K^2}{LJ}\omega + \frac{K}{LJ}V_a \quad ...(i)$$

$$\frac{d\omega}{dt} = 1.\frac{d\omega}{dt} + 0.\omega \quad ...(ii)$$

From equations *(i)* and *(ii)*, we get

$$\begin{bmatrix} \frac{d^2\omega}{dt^2} \\ \frac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -B/J & -K^2/LJ \\ 1 & 0 \end{bmatrix}\begin{bmatrix} \frac{d\omega}{dt} \\ \omega \end{bmatrix} + \begin{bmatrix} \frac{K}{LJ} \\ 0 \end{bmatrix} V_a$$

59. The characteristics equation is,

$$1 + GH = 0$$

$$s^3 + 6s^2 + 8s + k = 0$$

Constructing routh-Hurwitz array

s^3	1	8	0
s^2	6	k	
s^1	$k - 48$	–	
s	k		

The system will be just unstable

$k - 48 = 0$ or $k = \mathbf{48}$.

60.
$$\angle GH|_{\omega = 0.5a} = -\tan^{-1}\frac{\omega}{a}$$

$$= -\tan^{-1} 0.5 = 26.56°$$

From the plot,

at $\omega = 0$. $\angle GH = 22.5°$

Then error $= -22.50° (-26.56°) = 4.06°$

At $\omega = 0.5a$, $20 \log|GH|$

$= 20 \log k - 20 \log |1 + j0.5|$

$= 20 \log k - 0.97$

From the plot, $20 \log|GH| = 20 \log k$

Then, error in dB gain = **0.97 *d*B**

61. Forward path gain, $P_1 = \frac{2}{s^2}$

$$\mu\Delta_1 = 1$$

Individual loop gains = $\left(\frac{-3}{s}\right), \left(\frac{12}{s}\right), \left(\frac{18}{s^2}\right)$

P_{12} (Non-touching loop gain) $= 36/s^2$

Then, $T = \frac{P_1\Delta_1}{1 - P_{11} - P_{21} - P_{23} + P_{12}}$

$$= \frac{2./s^2}{1 + \frac{15}{s} + \frac{18}{s^2} + \frac{36}{s^2}} = \frac{2}{s^2 + 15s + 54}$$

$$= \frac{2}{(s+6)(s+9)} = \frac{1}{27\left(1+\frac{s}{6}\right)\left(1+\frac{s}{9}\right)}$$

62. Phase margin, $\phi = \angle G(j\omega) H(j\omega)|_{\omega=\omega_1} + 180°$

$$= -180° + 180° = 0°$$

[Since, at point $(-1, j0)$, $\angle G(j\omega)]. H(j\omega)|_{\omega=\omega_1} = 180°$

63. Initial value $= \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \dfrac{5}{s^2+3s+2} = 0$

64. Transfer function is derived as,

$$K_t \cdot \frac{d\theta}{dt} = e(t)$$

$$sK_t \cdot \theta(s) = E(s)$$

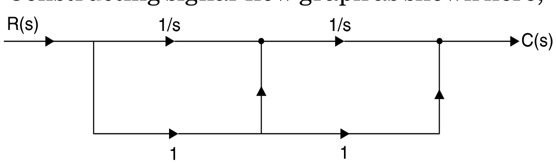
$$\therefore \quad \frac{E(s)}{\theta(s)} = sK_t$$

65. Constructing Routh-array,

s^3	1	1	0
s^2	−4	6	0
s^1	2	0	
s^0	6		

Number of sign changes in the first column is two, therefore the number of roots in the left half s-plane is 2.

66. Constructing signal-flow graph as shown here,



Number of forward paths is 3 having path gains,

$$P_1 = \frac{1}{s^2},$$

$$P_2 = 1,$$

$$P_3 = \frac{1}{s}$$

There are no loops, then, transfer function is given as,

$$T = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta} = \frac{1}{s^2} + 1 + \frac{1}{s}$$

$$= \frac{s^2+s+1}{s^2} \qquad [\Delta_1 = \Delta_2 = \Delta_3 = \Delta = 1]$$

67. Characteristic equation for this system is,

$$|sI - A| = 0$$

or
$$\begin{vmatrix} s & -2 \\ -2 & s \end{vmatrix} = 0$$

or
$$s^2 - 4 = 0$$

or
$$s = \pm 2.$$

68. $T(s) = \dfrac{C(s)}{R(s)} = \dfrac{k/s(s+2)}{1+\dfrac{k(1+sP)}{s(s+2)}} = \dfrac{k}{s^2+(2+Pk)s+k}$

Now, $\quad k = \omega_n^2,$

and $\quad (Pk+2) = 2\pi\omega_n$

Then, $\quad k = 25$

and $\quad P = 0.2.$

69. $C(s) = \dfrac{12.5 \times 8}{(s+6)^2+8^2} = \dfrac{100}{s^2+12s+100}$

Then, Output for unit step respoure

$$= \frac{1}{s} \cdot \frac{100}{s^2+12s+100}$$

and steady-state value $= \lim_{s\to 0} sC(s)$

$$= \lim_{s\to 0} \frac{100}{s^2+12s+100} = 1$$

70. $Y(s) = X(s) \cdot \dfrac{s}{s+1}$

$$Y(s) = \frac{1}{(s^2+1)} \cdot \frac{s}{s+1}$$

$$= \frac{A}{(s+j)} + \frac{B}{(s-j)} + \frac{C}{(s+1)}$$

$$= \frac{1}{2(1+j)}\left(\frac{1}{s-j}\right) + \frac{1}{2(1-j)(s+j)} - \frac{1}{2(s+1)}$$

Taking inverse Laplace transformation, we have

$$y(t) = \frac{1}{2(1+j)} e^{jt} + \frac{1}{2(1-j)} e^{-jt} - \frac{1}{2} e^{-t}$$

$$= \frac{1}{2}(\sin t + \cos t) - \frac{1}{2} e^{-t}$$

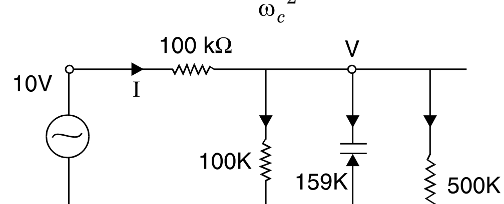
$$= \frac{1}{\sqrt{2}} \sin(t + 45°) - \frac{1}{2} e^{-t}$$

Then, steady state value $= \lim_{t\to\infty} y(t)$

$$= \frac{1}{\sqrt{2}} \sin(t + 45°).$$

71. The value of ω at which $|G(j\omega)| = 1$ is given by,

$$1 = \frac{\sqrt{(\omega_c a)^2 + 1}}{\omega_c^2} \qquad \text{.......... } (i)$$



Also $\angle G(j\omega)|_{\omega=\omega_c} + 180° = 45°$

or $\quad \tan^{-1}(\omega_c a) - 180° + 180° = 45°$

or $\quad \omega_c a = 1$

From equation (i), we have,

$$\omega_c = 2^{1/4} = 1.189 \text{ rad/sec.}$$

Then, $\quad a =$ **0.841.**

72. $|T(j\omega)| = \dfrac{|T(j\omega)^2 + 4|}{(j\omega+1)(j\omega+4)} = 0$

$\Rightarrow \quad -\omega^2 + 4 = 0,$

$\Rightarrow \quad \omega = 2$ rad/sec

73. $G(s)H(s) = \dfrac{K}{s^3}$

Characteristic equation is,

$1 + G(s)H(s) = 0$

$\therefore \quad s^3 + K = 0$

$\Rightarrow \quad \dfrac{dK}{ds} = 0$

$\Rightarrow \quad 3s^2 = 0$

$\Rightarrow \quad s = 0, 0$

Note : In all other options, all breaking points are not at origin.

74. $G(j\omega) = \dfrac{(j\omega+1)}{(j\omega)^2} = \dfrac{1+j\omega}{-\omega^2} = -\dfrac{1}{\omega^2} - \dfrac{1}{\omega}$

For Gain Margin, complex part of $G(j\omega) = 0$

$\Rightarrow \quad \dfrac{1}{\omega} = 0$

$\Rightarrow \quad \omega = \infty$

$\therefore$ Gain margin $= 20\log\dfrac{1}{|G(j\omega)|} = 20\log\dfrac{1}{\left|\dfrac{1}{\infty}\right|} = \infty$

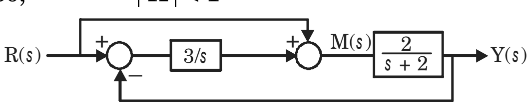
75. $1 + G(s)H(s) = 0$

$\Rightarrow \quad (1-K)s + (1+K) = 0$

$\Rightarrow \quad s'(1-K)\ 0 \quad 1-K > 0,\ 1+K > 0$

So, $\quad |K| < 1$

76.



$$M(s) = R(s) + [R(s) - Y(s)]\frac{3}{s}$$

and $\quad Y(s) = \dfrac{2}{s+2}\left[R(s)\left(1+\dfrac{3}{s}\right) - \dfrac{3}{s}Y(s)\right]$

$\therefore \quad \dfrac{Y(s)}{R(s)} = \dfrac{2(s+3)}{s^2+2s+6}$

$$E(s) = R(s) - Y(s) = \left[1 - \frac{2(s+3)}{s^2+2s+6}\right]$$

$$E(s) = R(s)\frac{s^2}{s^2+2s+6}$$

$$e_{ss} = \underset{s\to 0}{\text{Lt}}\, E(s) = 0$$

77. $G(s)H(s) = \dfrac{\pi e^{-0.25s}}{s}$

$$G(js)H(j\omega) = \frac{\pi[\cos(0.25\omega) - j\sin(0.25\omega)]}{j\omega}$$

$$= \frac{-\pi}{\omega}\sin(0.25\omega) - j\frac{\pi}{\omega}\cos(0.25\omega)$$

Imaginary part = 0

$\therefore \quad \dfrac{\pi}{\omega}\cos(0.25\omega) = 0$

$\Rightarrow \quad \dfrac{\omega}{4} = \dfrac{\pi}{2},\ \omega = 2\pi$

$\therefore \quad |G(j\omega)H(j\omega)|_{\omega = 2\pi}$

$$= \left|-\frac{\pi}{2\pi}\sin\left(\frac{2\pi}{4}\right)\right| = \left|-\frac{1}{2}\right| = 0.5$$

78. Here, $\quad T = \dfrac{Y(s)}{R(s)} = \dfrac{K + 0.366s}{s(s+1)}$

Since, $\quad$ P.M. $= 180° + \angle G(j\omega) + (j\omega)$

$$60° = 180° + \tan^{-1}\left(\frac{0.366\omega}{K}\right) - 90° - \tan^{-1}\omega$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{\frac{0.366\omega}{K} - \omega}{1 + \frac{0.366\omega}{K}}\right) = -30°$$

$$\Rightarrow \quad \frac{0.366\omega - \omega K}{K + 0.366\omega^2} = -\frac{1}{\sqrt{3}}$$

At cross-over frequency of 1 rad/sec,

we get $K + 0.366 = 1.732K - 0.634$

$\therefore \quad K = 1.366$

79. $P = \begin{bmatrix} 3 & -2 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$

The eigen-values corresponding to triangular matrix are $\lambda = 3, -2$ and 1.

For finding eigen vector, $[A - \lambda 1]\hat{x} = 0$

$$\begin{bmatrix} 3-\lambda & -2 & 2 \\ 0 & -2-\lambda & 1 \\ 0 & 0 & 1-\lambda \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda = -2$,

$$\begin{bmatrix} 5 & -2 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

or, $5x_1 - 2x_2 + 2x_3 = 0$

$x_3 = 0$

$3x_3 = 0$

If $\quad x_2 = k$ (Assume),

$5x_1 - 2k = 0,$

$x_1 = \dfrac{2}{5}k$

Then eigen vectors

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix}$$

80. $F(z) = \frac{1}{z+1} = \frac{z+1-z}{z+1} = 1 - \frac{z}{z-(-1)}$

$\therefore \quad z^{-1}[F(z)] = \delta(t) - (-1^n) \quad \left[\because z^{-1}\left(\frac{z}{z-a}\right) = a^n\right]$

81. $(sI - A)^{-1} = \begin{bmatrix} s & 1 \\ 0 & s+3 \end{bmatrix}^{-1} = \frac{\text{Adj}\,(sI - A)}{|\,sI - A\,|}$

$$= \frac{\begin{bmatrix} s+3 & -1 \\ 0 & s \end{bmatrix}}{\begin{vmatrix} s & 1 \\ 0 & s+3 \end{vmatrix}}$$

$$= \frac{\begin{bmatrix} s+3 & -1 \\ 0 & s \end{bmatrix}}{s(s+3)}$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{-1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}$$

$$\therefore \quad \phi(t) = L^{-1}(sI - A)^{-1} = \begin{bmatrix} 1 & \frac{1}{3}\left(1 - e^{-3t}\right) \\ 0 & e^{-3t} \end{bmatrix}$$

82. Zero state response = $\mathcal{L}^{-1}\phi(s)\,B\,U(s)$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{s} & \frac{-1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}\frac{1}{s}$$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{s^2} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} t \\ 0 \end{bmatrix}$$

State transition equation = Zero input response + zero state response.

$$\therefore \quad X(t) = \phi(X)\,X(0) + t$$

$$= \begin{bmatrix} -1 + 1 - e^{-3t} \\ 0 + 3e^{-3t} \end{bmatrix} + \begin{bmatrix} t \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} t - e^{-3t} \\ 3e^{-3t} \end{bmatrix}$$

83. It is state space representation using phase variable.

$$\text{Standard form} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_n & -a_{n-1} & -a_{n-2} \end{bmatrix}$$

Thus ATQ $\quad a_n = 1, a_{n-1} = -2, a_{n-2} = 4$

85. $H(j\omega) = \frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$

$$= \frac{10^4(1+s)}{10\left(1+\frac{s}{10}\right)(100)^2\left(1+\frac{s}{200}\right)^2}$$

$$= \frac{0.1\,(1+s)}{\left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)^2}$$

$\therefore \quad$ Value of $\quad K = +20\log_{10}(0.1)$

$= -20$ db.

So plot will start from –20 db ft .

87. $F(s) = s^5 - 3s^4 + 5s^3 - 7s^2 + 4s + 20$

We can solve it by making Routh Hurwitz array.

s^5	1	5	4
s^4	–3	–7	20
s^3	8/3	20/3	0
s^2	5	20	0
s^1	0	0	0
s^0	20	0	0

We can replace 1st element of s^1 by 10.

If we observe 1st column, sign is changing two times, so we have two poles on right half side of imaginary axis and $5s^2 + 20 = 0$.

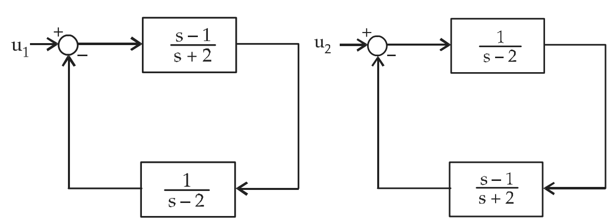
So, $s = \pm 2j$ and 1 pole on left side of imaginary axis.

88. If GH plot incircles $(-1, j0)$ that is the critical point, then the system becomes unstable. So option 1 is there which does not enclose the $(-1, j0)$ other all are incircling the critical point.

89. Transfer function for u_1

$$TF_1 = \frac{(s-1)/(s+2)}{1 + \left(\frac{s-1}{s+2}\right).\left(\frac{1}{s-1}\right)}$$

$$= \frac{(s-2)}{(s+3)}$$



Hence stable

Transfer function for u_2

$$TF_2 = \frac{\left(\frac{1}{s-1}\right)}{1+\left(\frac{1}{s-1}\right)\cdot\left(\frac{s-1}{s+2}\right)}$$

$$= \frac{1}{(s+3)(s-1)}$$

Hence unstable, as it has pole at right side of *s*-plane.

90. Given, $G(s) = \frac{1}{s(s+1)(s+2)}$

$$G(j\omega) = \frac{1}{j\omega(1-j\omega)(2+j\omega)} = \frac{-j(1-j\omega)(2-j\omega)}{\omega(1-\omega^2)(4+\omega^2)}$$

$$= \frac{j(2-3j\omega-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)} = \frac{-3\omega-j(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$= \frac{-3}{(1+\omega^2)(4+\omega^2)} - j\frac{(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$

$$\Rightarrow \quad x = \frac{3}{(1+\omega^2)(4+\omega^2)},$$

$$y = -\frac{(2-\omega^2)}{\omega(1+\omega^2)(4+\omega^2)}$$

At $\omega \to 0, x \to \frac{3}{4}, y \to -\infty$

92. $G(z) = z^{-1} + z^{-2}$

Characteristic equation is

$$1 + KG(z) = 0$$

$$\Rightarrow 1 + K(z^{-1} + z^{-2}) = 0$$

$$\Rightarrow z^2 + Kz + I = 0$$

Using stability criteria,

$$|K| < 1$$

i.e. $-1 < K < 1$

93. Characteristic equation is

$$1 + \frac{K(s+3)}{(s+8)^2} = 0$$

$$\Rightarrow s^2 + (16 + K)s + 3K + 64 = 0$$

Routh's array,

s^2	1	3K + 64
s	16 + K	
s^0	3K + 64	

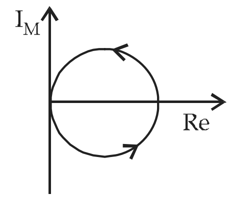
$\Rightarrow$ No such K exist to make all element of a row equal to 0.

94. By block diagram, technique reduction method

95. $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

$$Y = \frac{1}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

By rationalization and varying frequency ω, then locus is



96. Steady state value of z

$$= \underset{s\to 0}{\ell t}\ sE(s)\frac{k_i}{s}$$

$$= \underset{s\to 0}{\ell t} \frac{s.\frac{1}{s}.\frac{k_i}{s}}{\left[1+\left(k_p+\frac{k_i}{s}\right)\left(\frac{\omega^2}{s^2+2s\omega_r s+\omega^2}\right)\right]}$$

$$= \underset{s\to 0}{\ell t} \frac{k_i/s}{1+\left(k_p+\frac{k_i}{s}\right)\left(\frac{\omega^2}{s^2+2s\omega_r s+\omega^2}\right)}$$

$$= 1$$

97. By Clayey – Hamilton theorem,

Every square matrix satisfies its own characteristic equation

$$\alpha(\lambda) = \lambda^3 + \lambda^2 + 2\lambda + 1 = 0$$

$$\alpha(P) = P^3 + P^2 + 2P + I = 0$$

$$\Rightarrow \quad I = -P^3 + P^2 + 2P$$

Premultiplying by p^{-1}, we get

$$P^{-1} = -[P^2 + P + 2I]$$

98. $G(s) = \frac{y(s)}{U(s)} = \frac{1}{s^2+3s+2}$

$$\therefore \quad y(s) = \frac{1}{s^2+3s+2}U(s)$$

But $\quad U(t) = \delta(t-1)$

and $\quad U(s) = e^{-s}$

$$\therefore \quad y(s) = \frac{e^{-s}}{s^2+3s+2}$$

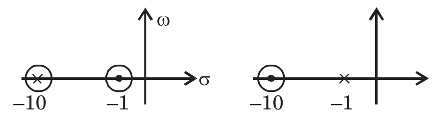
For steady state,

$$\underset{t\to\infty}{y(t)} = \underset{s\to 0}{Lt}\ sy(s)$$

$$= \underset{s\to 0}{} \frac{e^{-s}}{s^2+3s+2}$$

$$= \frac{1}{2} = 0.5$$

99. $C_1 = \frac{10(s+1)}{(s+10)}$ $C_2 = \frac{s+10}{10(s+1)}$



zero dominates pole ∴ lead compensator

pole dominates zero ∴ lag compensator

Hence C_1 is lead compensator and C_2 is lag compensator.

Alternately

$$\angle\phi \text{ for } G = \tan^{-1}\omega - \tan^{-1}\frac{\omega}{10}$$

or $$\angle\phi \text{ for } G = \tan^{-1}\frac{\omega}{10}\ \omega - \tan^{-1}\omega$$

Hence C_1 is lead compensator and C_2 is lag compensator.

100. Transfer function of given plot is of the form

$$T(s) = \frac{k(1+sT_1)(1+sT_2)}{s^2}$$

so, it will have two poles and two zeroes.

First – 40 dB/decade slope indicates 2 pole in system.

Next – 20 dB/decade slope indicates 1 zero is added.

Next – dB/decade slope indicates 1 more zero is added.

Alternately

At $\omega = 0.1$, slope changes from – 40 dB/decade to – 20 dB/decade, then, zero at $\omega = 0.1$

At, $\omega > 0.1$, slope changes from – 20dB/decade to 0 dB/decade, then one more zero at $\omega > 0.1$

Since, for $\omega < 0.1$, slope is – 40 dB/decade there is alveag two poles.

Hence (c) is the correct option.

101. $K > 0$

Characteristic equation of the system is

$$1 + G(s)H(s) = 0$$

$$\Rightarrow \quad 1 + \frac{K}{s(s+3)(s+10)} = 0$$

$$\Rightarrow \quad s(s^2 + 13s + 30) + K = 0$$

$$\Rightarrow \quad s^3 + 13s^2 + 30s + K = 0$$

Routh array

s^3	1	30
s^2	13	K
s^1	$\frac{390-K}{13}$	0
s^0	K	

For system to be stable, all element of first column must be positive.

$$\therefore \quad \frac{390-K}{13} > 0$$

$\Rightarrow \quad K < 390°$

and $\quad K > 0$

Hence range of K is $0 < K < 390$

102. $$M(s) = \frac{100}{s^2 + 20s + 100}$$

Comparing with standard form,

$$M(s) = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

we get $\quad 2\xi\omega_n = 20$

$$\Rightarrow \quad \xi = \frac{20}{2\times 10} = 1$$

and $\quad \omega_n^{\,2} = 100$

$\Rightarrow \quad \omega_n = 10$

For $\xi = 1$, system is critically damped.

103. The relative error of product or division of different quantities is equal to the sum of relative errors of individual quantities.

105. Two roots at $s = \pm j$ and one in left halfs-plane

s^3	2	2
s^4	4	4

Characteristic equation is

$$A(s) = 2s^3 + 4s^2 + 2s + 4 = 0$$

$\Rightarrow s^3 + 2s^2 + s + 2 = 0$

$\Rightarrow (s+2)\ (s^2+1) = 0$

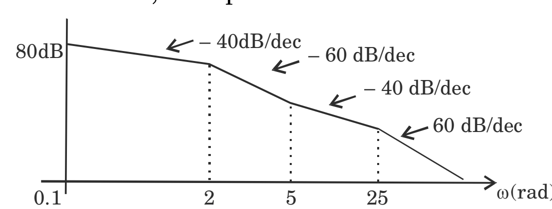
$\Rightarrow s = -2, \quad s = \pm j$

106. At ω = 0.1, 2 poles

At ω = 2,1 pole

At ω = 5, 1 zero

At ω = 25, 1 pole



So, transfer function,

$$T(s) = \frac{k(s+5)}{s(s+2)(s+25)}$$

$$\therefore \quad 80 = 20\log\frac{5k}{(0.1)^2 \times 2 \times 25}$$

$$\Rightarrow \quad 4 = \log\frac{5k}{0.5}$$

$\Rightarrow \quad 10k = 10^4$

$\Rightarrow \quad k = 10^3 = 1000$

$$\therefore \quad T(s) = \frac{1000(s+5)}{s^2(s+2)(s+25)}$$

107. Given : $G(s) = \frac{K}{(s+1)(s+2)}$

From curve, steady state error = 1 – 0.75 = 0.25

$$\therefore \quad E(s) = \frac{R(s)}{1+H(s)G(s)} = \frac{1/s}{1+1.\frac{k}{(s+1)(s+2)}}$$

$$\therefore \quad e_{ss} = \lim_{s\to 0} sE(s)$$

$$= \lim_{s\to 0} s.\frac{1/s}{1+\frac{k}{(s+1)(s+2)}} = 0.25$$

$$\Rightarrow \quad \frac{1}{1+\frac{k}{2}} = 0.25$$

$$\Rightarrow \quad 1 = 0.25 + \frac{0.25k}{2}$$

$$\Rightarrow \quad k = \frac{0.75\times 2}{0.25} = 6$$

108. Given : $G(s) = \frac{e^{-0.1s}}{s}$

$$\Rightarrow \quad G(j\omega) = \frac{e^{-0.1j\omega}}{(j\omega)}$$

Phase crossover frequency :

$$-90 - 0.1\,\omega \times \frac{180}{\pi} = -180$$

$$\Rightarrow \quad \frac{18}{\pi}\omega = 90$$

$$\Rightarrow \quad \omega = 5\pi = 15.7 \text{ rad/sec}$$

Gain margin :

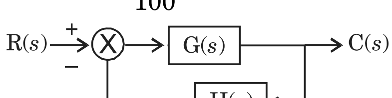
$$a = |G(j\omega)|_{\omega = 18.7} = \frac{1}{\omega} = \frac{1}{15.7}$$

$\therefore$ Gain margin = 20 log a

$$= 20 \log\left(\frac{1}{15.7}\right) = -23.9 \text{ dB.}$$

109. Gain, $G(s) = 100 \pm 10\%$

$$H(s) = \frac{9}{100}$$



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

When $G(s) = 100 + 10 = 110$,

$$\frac{C(s)}{R(s)} = \frac{110}{1+110\times\frac{9}{100}} = \frac{1100}{109}$$

$= 10.091 = 10 + 0.1$

$= 10 + 1\%$ of $10 = 10.1$

When $G(s) = 100 - 10 = 90$,

$$\frac{C(s)}{R(s)} = \frac{90}{1+90\times\frac{9}{100}} = \frac{900}{91}$$

$= 9.89 \approx 9.9$

$= 10 - 0.1 \neq 10 - 1\%$ of 10

Hence overall system gain = $10 \pm 1\%$

110. Given : $G(s) = \frac{2}{s+1} = \frac{A}{Ts+1}$

$\therefore$ Time constant, T = 1

For 98%, time required = 4T = 4 sec

$$\therefore \quad \frac{C(s)}{R(s)} = \frac{2}{s+1}$$

$$\therefore \quad C(s) = \frac{2}{(s+1)}\cdot\frac{1}{s}$$

$$\Rightarrow \quad C(s) = 2\left[\frac{1}{s} - \frac{1}{s+1}\right]$$

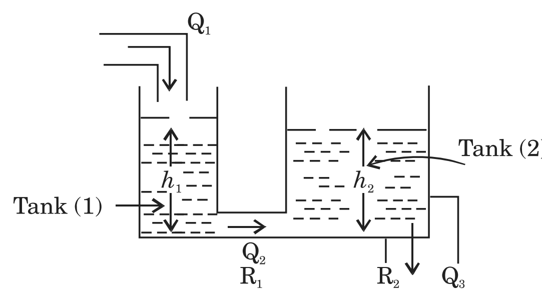
$$\Rightarrow \quad C(t) = 2\left[1-e^{-t}\right] = 2\times 0.98$$

$$\Rightarrow \quad 1 - e^{-t} = 0.98$$

$$\Rightarrow \quad e^{-t} = 0.02$$

$$\Rightarrow \quad t = \text{-}\ln 0.2 = 3.91 \text{ sec}$$

111.



C_1 = capacity of tank (1)

C_2 = capacity of tank (2)

h_1, h_2 = heights of water

Q_1, Q_2 = flow rates

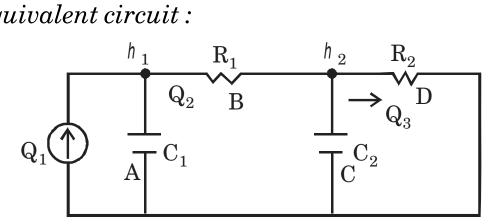
R_1, R_2 = resistances of flow

$$Q_1 = Q_2 + C_1\frac{dh_1}{dt}$$

$$Q_2 = Q_3 + C_2\frac{dh_2}{dt}$$

$$\therefore \quad Q_2 = \frac{h_1 - h_2}{R_1},\; Q_3 = \frac{h_2}{R_2}$$

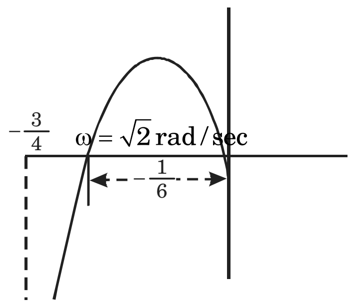
Equivalent circuit :



$\therefore$ A, C = capacitance

B, D = resistances

112.
$$G(s) = \frac{1}{s(s+1)(s+2)}$$
$$G(j\omega) = \frac{1}{j\omega(j\omega+1)(j\omega+2)}$$
$$M = \frac{1}{\omega\sqrt{\omega^2+1}\sqrt{\omega^2+4}}$$
$$\angle\phi = -90 - \tan^{-1}\omega - \tan^{-1}\frac{\omega}{2}$$



For $\omega = 0$, $M = \infty$; $\angle\phi = -90$

For $\omega = \infty$, $M = 0$; $\angle\phi = -90 - 90 - 90 = -270$

So Cutting Real Axis

Imaginary part of $G(j\omega) = 0$

i.e. $\text{Img}\{G(j\omega)\} = 0$

$$\Rightarrow \quad \text{Im}\left\{\frac{1}{j\omega(-\omega^2+3)\omega+2}\right\} = 0$$
$$\Rightarrow \quad \text{Im}\left\{\frac{1}{-j\omega^3 - 3\omega^2 + 2j\omega}\right\} = 0$$
$$\Rightarrow \quad \text{Im}\left\{\frac{1}{-3\omega^2 + j(2\omega - w^3)}\right\} = 0$$
$$\Rightarrow \quad \text{Im}\left\{\frac{-3\omega^2 - j(2\omega-\omega^3)}{(3\omega^2)^2 + (2\omega - w^3)^2}\right\} = 0$$
$$\therefore \quad 2\omega - \omega^3 = 0$$
$$\Rightarrow \quad \omega^2 = 2 \cdot \omega = 0$$

Neglecting $\omega = 0$, we have

$$\omega = \sqrt{2}$$
$$\therefore \quad M\Big|_{at\ \omega=\sqrt{2}} = \frac{1}{\sqrt{2}\sqrt{3}\sqrt{2+4}}$$
$$= \frac{1}{\sqrt{36}} = \frac{1}{6} < \left(\frac{3}{4}\right)$$

113. Given : $\dot{x} = Ax + Bu$;

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}, \ B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

For stability, characteristic equation

$$= \left|\left[(sI - A)^{-1}\right]\right| = \left|\begin{bmatrix} s+1 & 2 \\ 0 & s-2 \end{bmatrix}^{-1}\right|$$
$$= \frac{1}{M}\begin{vmatrix} (s-2) & 2 \\ 0 & s+1 \end{vmatrix} = 0$$
$$\therefore \quad M = (s+1)\times(s-2) = 0$$
$$\Rightarrow \quad (s-2)(s+1) = 0$$
$$\Rightarrow \quad s = 2, s = -1$$

So one root in right hand side of s-plane, so system is unstable.

For controllability : $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$AB = \begin{bmatrix} -1 & 2 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$
$$\left|[B : AB]\right| = \left|\begin{bmatrix} 0 & 2 \\ 1 & 2 \end{bmatrix}\right|$$
$$= 0 - 2 = -2 \neq 0$$

So controllable.

114. Given : $s(s+1)(s+3) + k(s+2) = 0$; $k > 0$

$$\Rightarrow \quad 1 + \frac{k(s+2)}{s(s+1)(s+3)} = 0$$

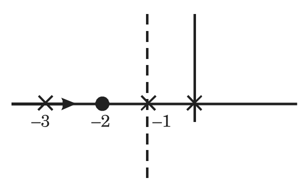
But $1 + G(s)H(s) = 0$

$$\therefore \quad G(s)H(s) = \frac{k(s+2)}{s(s+1)(s+3)}$$

Roots $s = 0$, $s = -1$, $s = -3$ (poles) ; $s = -2$ (zero)

It has real pole or zeros.

$$\phi_A = \frac{2q+1}{n-m}\times 180°$$



At $s = 0$, Asymptotes

$$\phi_0 = \frac{(2*0+1)180}{2} = 90°$$

At $s = 1$, $$\phi_1 = \frac{(2*1+1)*180}{2} = 270°$$

Centroid,

$$(-\sigma_A) = \frac{\Sigma\text{real parts of poles} - \Sigma\text{real poles of zeros}}{\text{number of poles} - \text{number of zeros}}$$
$$= \frac{-1-3+2+0}{2} = -1$$
$$\therefore \quad Re[s] = -1$$

115. At $\angle G(j\omega) = -180$, magnitude $M = 0.5$

$$\therefore \quad G.M = 20\log\left(\frac{1}{0.5}\right) = 6\text{dB}$$

At $|G(j\omega)| = 1$, phase angle $\angle G(j\omega) = -150$

$$\therefore \quad P.M = 180 + (-150) = 30°$$

116. For step input, $e_{ss} = 0.1 = \frac{1}{1+k}$

$$\Rightarrow \quad K = 9$$
$$G(S) = \frac{9}{S+1}$$

Now input is pulse $r(t) = 10\,[u(t) - u(t-1)]$

$$R(S) = 10\left[\frac{1-e^{-s}}{s}\right]$$

$$e_{ss} = \underset{S\to 0}{\text{Lt}} \frac{S.R(s)}{1+G(S)H(S)}$$

$$= \underset{S\to 0}{\text{Lt}} \frac{\dfrac{S\,10[1-e^{-s}]}{S}}{\dfrac{S+10}{S+1}} = \frac{0}{10} = 0$$

117. $|Z| < 1$, so $|Y| > 1$

Z is having +ve real part and positive imaginary part (from characteristics)

So, Y should have +ve real part and negative imaginary part.

118. Since, open loop system stability is depends only on pole locations, hence system is stable.

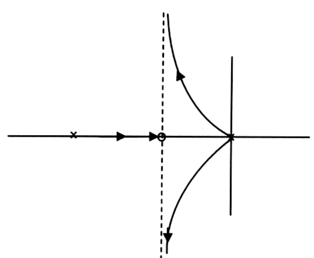
There is one zero on right half of s-plane so system is non-minimum phase.

119. $F_2(t) = L\{f(t-\tau)\} = e^{-S\tau} F_1(S)$

$$G(S) = \frac{e^{-s\tau}F_1(s).F_1^*(s)}{|F_1(s)|^2} = e^{-s\tau}$$

$G(t) = L^{-1}\{G(S)\} = \delta(t-\tau)$

120. Centroid, $\sigma = \dfrac{-2-\left(-\dfrac{2}{3}\right)}{3-1} = \dfrac{-6+2}{6} = \dfrac{-4}{6} = -\dfrac{2}{3}$



$$\text{Asymptotes} = \frac{(29\pm 1)180}{p-z}$$

$\therefore \quad \theta_1 = \dfrac{180}{2} = 90°$

and $\quad \theta_2 = \dfrac{18\times 3}{2} = 270°$

121. By Dolittle's decomposition,

$$\begin{bmatrix}2 & 1\\ 4 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\ l_{21} & 1\end{bmatrix}\begin{bmatrix}u_{11} & u_{12}\\ 0 & u_{22}\end{bmatrix}$$

We have $\quad u_{11} = 2, u_{12} = 1$

$l_{21}u_{11} = 4$

$\Rightarrow \quad l_{21} = 2$

and $\quad l_{21}u_{12} + u_{22} = -1$

$\Rightarrow \quad u_{22} = -1 - l_{21}u_{12} = -3$

then, $\begin{bmatrix}2 & 1\\ 4 & -1\end{bmatrix} = \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}2 & 1\\ 0 & -3\end{bmatrix}$ ...(A)

eqn (A) does not signfy any choice

Now, from crout's decomposition,

$$\begin{bmatrix}2 & 1\\ 4 & -1\end{bmatrix} = \begin{bmatrix}l_{11} & 0\\ l_{21} & l_{22}\end{bmatrix}\begin{bmatrix}1 & u_{12}\\ 0 & 1\end{bmatrix}$$

We get, $\quad l_{11} = 2, l_{11}u_{12} = 1$

$$u_{12} = \frac{1}{2} = 0.5$$

and, $\quad l_{21} = 4, l_{21}u_{12} + l_{22} = -1$

$\Rightarrow \quad l_{22} = -3$

Then, $\begin{bmatrix}2 & 1\\ 4 & -1\end{bmatrix} = \begin{bmatrix}2 & 0\\ 4 & -3\end{bmatrix}\begin{bmatrix}1 & 0.5\\ 0 & 1\end{bmatrix}$

Hence the choice is (*d*).

122. $\dfrac{Y(s)}{R(s)} = \dfrac{1}{s^2+(k+1)s+1}$

$2\xi\,\omega_n = k+1$

$\Rightarrow \xi = \left(\dfrac{k+1}{2}\right);$

Peak over shoot $= e^{-\pi\xi/(1-\xi^2)}$

123. $$|G(s)| = \frac{(9-\omega^2)\sqrt{4+\omega^2}}{\sqrt{\omega^2+1}\sqrt{\omega^2+9}\sqrt{16+\omega^2}} = 0$$

$\therefore \quad \omega^2 = 9$

$\Rightarrow \quad \omega = 3$ rad/s

124. $L[f(t)] = F(s) = \dfrac{1}{s^2+s+1};$

$$L[tf(t)] = (-1)\frac{dF(s)}{ds}$$

$$= (-1)\left[\frac{-(2s+1)}{s^2+s+1}\right] = \frac{2s+1}{s^2+s+1}$$

125. $Q_c = [B\ AB\ A^2B]$

$$A = \begin{bmatrix}0 & a_1 & 0\\ 0 & 0 & a_1\\ a_3 & 0 & 0\end{bmatrix}; B = \begin{bmatrix}0\\0\\1\end{bmatrix};$$

$$AB = \begin{bmatrix}0\\ a_2\\ 0\end{bmatrix}; A^2B = \begin{bmatrix}a_1a_2\\ 0\\ 0\end{bmatrix}$$

$$Q_c = \begin{bmatrix}0 & 0 & a_1a_2\\ 0 & a_2 & 0\\ 1 & 0 & 0\end{bmatrix}$$

If rank of $Q_c = 3$ = order of matrix, then

Q_c is controllable $\left.\begin{matrix}a_1 \neq 0\\ a_2 \neq 0\\ a_3 \neq 0\end{matrix}\right\}$, then $|Q_c| \neq 0$

126. Characterisitic equation is

$$1 + G(s)H(s) = 0$$

$$\therefore \quad 1 + \frac{k(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$$\Rightarrow \quad s^3 + s^2 a + s(k+2) + (k+1) = 0$$

s^3	1	$k+2$
s^2	a	$k+1$
s^1	$\frac{(k+2)a-(k+1)}{a}$	
s^0		

$as^2 + (k+1) = 0$; $s = j\omega$; $s^2 = -\omega^2$; $\omega = 2$

$-a\omega^2 + (k+1) = 0$; $a\omega^2 = k+1$; $4a = k+1$;

From options, $k = 2$, $a = 0.75$

127. $y(t) = \int_{-\infty}^{t} x(\tau)\cos(3\tau)d\tau$

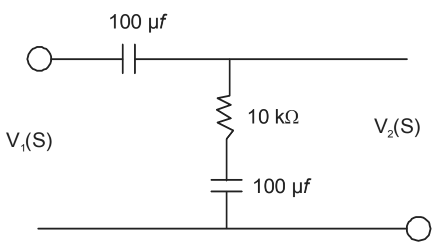
Since $y(t)$ and $x(t)$ are related with some function of time, so they are not time-invariant.

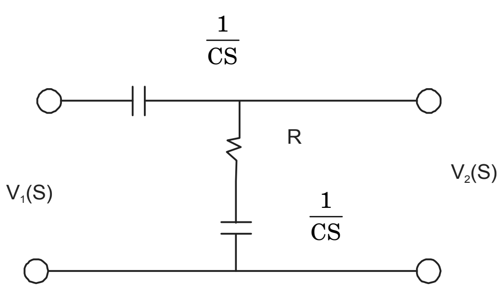
Let $x(t)$ be bounded to some finite value k.

$$y(t) = \int_{-\infty}^{t} K\cos(3\tau)d\tau < \infty$$

$y(t)$ is also bounded. Thus system is stable.

128.





By voltage division rule

$$v_2(s) = \frac{R + \frac{1}{cs}}{R + \frac{1}{cs} + \frac{1}{cs}} V_1(s)$$

$$\frac{v_2(s)}{v_1(s)} = \frac{10\times10^3 + \frac{1}{100\times10^{-6}s}}{10\times10^3 + \frac{2}{100\times10^{-6}s}}$$

$$= \frac{10^4 + \frac{10^4}{s}}{10^4 + 11}$$

$$\frac{v_2(s)}{v_1(s)} = \frac{(s+1)}{(s+2)}$$

129.

$$\frac{y(s)}{u(s)} = \frac{1}{s}$$

$$y(s) = \frac{1}{s} \times u(s)$$

For unit step $1/s$

$$u(s) = \frac{1}{s}$$

$$y(s) = \frac{1}{s^2}$$

$$y(t) = t\,u(t)$$

130. As $h(t) = t\,u(t)$

input   response

$\delta(t) \rightarrow t\,a\,(t)$

$$u(t) \rightarrow tu(t)dt = u = \int_0^t tdt = \frac{t^2}{2}u(t)$$

$$u(t-1) \rightarrow \frac{(t-1)^2}{2}u(t-1)$$

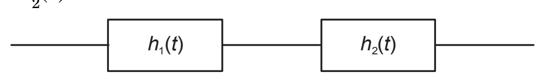
131. Consider option (*a*): In which all the poles lie on the left of $j\omega$ axis which satisfy casual stable LTI system.

Option (*b*): For a stable casual system, there are no restriction for the position of zeroes on s plane.

Option (*c*): text true.

Option (*d*): Roots of characteristic equation are all closed loop poles and they all line on the left side of the $j\omega$ axis.

132. Two systems with impulse responses $(h_1)(t)$ and $h_2(t)$ are connected in cascade



Then the overall impulse response of the Cascaded system in given by convolution $h_1(t)$ and $h_2(t)$

Overall Impulse Respone = Convolution of $h_1(t), h_2(t)$

= Product of $H_1(s)$, $H_2(s)$

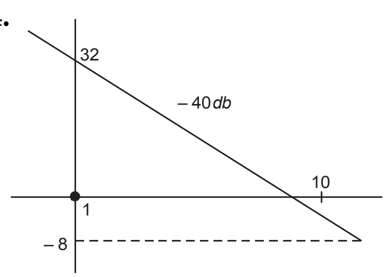
Where $H_1(s)$, $H_2(s)$ are transfer function is S domain.

133. For max power

$$R_L = \sqrt{R_s^2 + X_s^2}$$

$$= \sqrt{4^2 + 3^2} = 5$$

134.



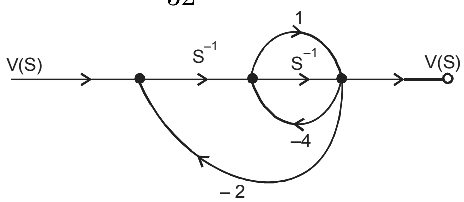
$\Rightarrow \qquad 20 \log k = 32$

$k = 10^{1.6} = 39.8$

$\Rightarrow$ As slope is – 40db/decade so two pols at ons'm

so $\qquad T(s) = \dfrac{39.8}{52}$

135.



Forward path

$P_1 = S^{-1}, S^{-1} = S^{-2} \quad | \quad \Delta_1 = 1$

$P_2 = S^{-1} \quad | \quad \Delta_2 = 1$

Loop

$L_1 = -4S^{-1}$

$L_2 = -2S^{-1}S^{-1} = -2S^{-2}$

$L_3 = -2S^{-1}$

$L_4 = -4$

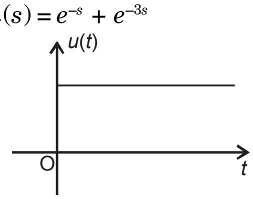
$$T(S) = \frac{S^{-2}+S^{-1}}{1+4+2S^{-1}+2S^{-2}+4S^{-1}}$$

$$= \frac{S^{-1}+S^{-2}}{S+6S^{-1}+2S^{-2}}$$

$$= \frac{S+1}{5S^2+6S+2}$$

136. Apply Laplace transform

$h(s) = e^{-s} + e^{-3s}$



for input step voltage $\rightarrow$

$$y(s) = h(s)\frac{1}{s}$$

$$= \left[e^{-s}+e^{-3s}\right]1/s$$

$y(t) = u(t-1) + u(t-3)$

$u(t) = 1$ for t 30 = 0 prt < 0

$\therefore$ O/P in $y(2)\ u(t-1) + u(t-3) = (4-1)$

$= 4(z-1) + 4\ (2-3)$

$= 4(1) + (4-1)$

$= 0 + 1 = 1$

137. When all elements of row have zero values which leads to auxiliary equation. This premature termination of the array indicates the presence of imaginary roots.

138. $\dfrac{C(s)}{R(s)} = \dfrac{k}{(s+1)(s+2)-k}$ will give the root locus given the diagram.

139. State transition matrix,

$\phi(t) = \mathcal{L}^{-1}\,[sI - A]^{-1}$

$$= \mathcal{L}^{-1}\begin{bmatrix} s-1 & 0 \\ -1 & s-1 \end{bmatrix}^{-1}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{(s-1)^2}\begin{bmatrix} s-1 & 0 \\ 1 & s-1 \end{bmatrix}\right\}$$

$$\phi(t) = \mathcal{L}^{-1}\begin{bmatrix} \dfrac{1}{s-1} & 0 \\ \dfrac{1}{(s-1)^2} & \dfrac{1}{s-1} \end{bmatrix} = \begin{bmatrix} e^t & 0 \\ te^t & e^t \end{bmatrix}$$

140. $$G(s) = \frac{5(s+4)}{s(s+0.25)(s^2+4s+25)}$$

$$G(j\omega) = \frac{5(j\omega+4)}{j\omega(j\omega+0.25)[(j\omega)^2+4j\omega+25]}$$

$$= \frac{5\times4\times4\left(\dfrac{j\omega}{4}+1\right)}{j\omega\ (4j\omega+1)\left[\left(\dfrac{j\omega}{5}\right)^2+\dfrac{4}{25}j\omega+1\right]\times 25}$$

$$= \frac{80}{25}\,\frac{\left(\dfrac{j\omega}{4}+1\right)}{j\omega(4j\omega+1)\left[\left(\dfrac{j\omega}{5}\right)^2+\dfrac{4(j\omega)}{25}+1\right]}$$

Constant gain term = $\dfrac{80}{25}$ = 3.2

Corner frequencies are $\omega = 4$, $\omega = 0.25$, $\omega = 5$

Then highest corner frequency $\omega = 5$ rad/sec.

141. For controllability,

$M_C = [Q\ \ PQ]$

$$M_C = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$$

$$\Delta M_C = \begin{vmatrix} 0 & 1 \\ 1 & -3 \end{vmatrix} \neq 0$$

Therefore, the system is Controllable.

Now, for observability.

$$M_o = \left[R^T\ R^Tp^T\right] = \begin{bmatrix} 0 & 0 \\ 1 & -3 \end{bmatrix}$$

$$\Delta M_o = \begin{vmatrix} 0 & 0 \\ 1 & -3 \end{vmatrix} = 0$$

Therefore, the system is not observable.

142. From the signal flow graph, G(s) = $\dfrac{C(s)}{U(s)}$

By mason's gain relation,

Transfer function = $\dfrac{P_1\Delta_1 + P_2\Delta_2 + \ldots..}{\Delta}$

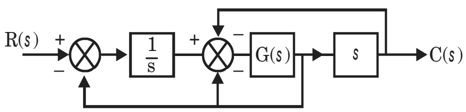
$$P_1 = \frac{h_1}{S} \; ; \; P_2 = \frac{h_0}{s_2}$$

$$\Delta_1 = \left[1 + \frac{a_1}{s}\right]; \; \Delta_2 = 1;$$

$$\Delta = 1 + \frac{a_1}{s} + \frac{a_0}{s^2}$$

$$\text{Transfer function} = \frac{\frac{h_1}{s}\left[1 + \frac{a_1}{s}\right] + \frac{b_0}{s^2}}{1 + \frac{a_1}{s} + \frac{a_0}{s^2}} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$$

144.



If G(s) = S

$$\frac{C(s)}{R(s)} = \frac{S}{s^2 + s + 2}$$

145. Transfer function

$$\Rightarrow C[SI - A]^{-1}.B. = [1 \quad 0]\begin{bmatrix} S & -1 \\ 1 & (s+1) \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\text{Transfer function} = \frac{1}{s^2 + s + 1}$$

$$\frac{G(s)}{1 + G(s)} = \frac{1}{s^2 + s + 1}$$

$$\Rightarrow \qquad G(s) = \frac{1}{s^2 + s}$$

Steady state error for unit step

$$e_{ss} = \frac{A}{1 + K_p} = \frac{1}{1 + \lim_{s \to 0} G(s)}$$

$$e_{ss} = \frac{1}{1 + \lim_{s \to 0} \frac{1}{s^2 + s}} = \frac{1}{1 + \infty}$$

$$e_{ss} = 0$$

146.

$$G(s) = k.\frac{(1 + 3s)}{(1 + s)}$$

$$G(s) = \frac{3k.\left(s + \frac{1}{3}\right)}{(s + 1)}$$

Here $k = 1$

$$\frac{1}{T} = \frac{1}{3} \Rightarrow \frac{1}{\alpha T} = 1$$

$$\omega_m = 1/\sqrt{3}; \alpha = 1/3$$

$$G(s)\Big|_{\omega = 1/\sqrt{3}} = \frac{\sqrt{4}}{\sqrt{4/3}} \Rightarrow \sqrt{3}$$

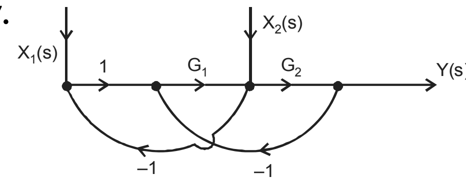
$$G_m\Big|_{\text{in dB}} = 20 \log \sqrt{3} = 4.77 \text{ dB}$$

$$\phi_m = \sin^{-1}\left[\frac{1 - \alpha}{1 + \alpha}\right]$$

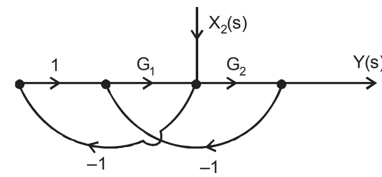
$$\alpha = 1/3 = \sin^{-1}\left[\frac{1 - 1/3}{1 + 1/3}\right] = \sin^{-1}\left(\frac{1}{2}\right)$$

$$\phi_m = 30°$$

147.



when $X_1(s) = 0$



Forward path gain,

$$P_1 = G_2$$

Loop gain,

$$L_1 = -G_1$$

$$L_2 = -G_1 G_2$$

$$\Delta = 1 - (L_1 + L_2) + L_1 L_2$$

$$= 1 + G_1 + G_1 G_2 \qquad \Big| L_1 L_2 = 0$$

$$\Delta = 1 + G_1(1 + G_2)$$

Transfer function,

$$\frac{Y(s)}{G_2(s)} = \frac{P_1 \Delta_1}{\Delta}$$

$$= \frac{G_2.1}{1 + G_1(1 + G_2)} \qquad \Big| \Delta_1 = 1$$

$$\frac{Y(s)}{G_2(s)}\Bigg|_{G_1(s) = 0} = \frac{G_2}{1 + G_1(1 + G_2)}$$

148. Initial slope is zero

It is type '0' system

(No pole at origin)

Starting gain = 20 log K db

20 log K = 20

K = 10

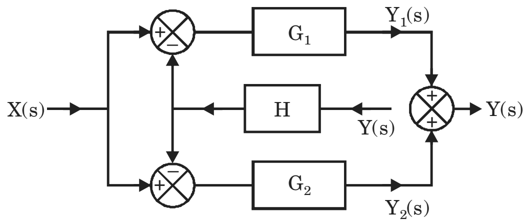
Given T/F

$$G(s) = \frac{s + 1000}{10(s + 10)}$$

$$= \frac{1000\left(1 + \frac{s}{1000}\right)}{10 \times 10\left(1 + \frac{s}{10}\right)}$$

$$G(s) = \frac{10\left(1 + \frac{s}{1000}\right)}{\left(1 + \frac{s}{10}\right)}$$

150.



$$Y(s) = Y_1(s) + Y_2(s)$$
$$Y_1(s) = (X - HY)G_1 = XG_1 - HG_1Y$$
$$Y_2(s) = (X - HY)G_2 = XG_2 - HG_2Y$$
$$Y(s) = XG_1 - HG_1Y + XG_2 - HG_2Y$$
$$Y = X(G_1 + G_2) - H(G_1 + G_2)Y$$
$$Y + Y(G_1 + G_2)H = (G_1 + G_2)X$$
$$Y(1 + (G_1 + G_2)H) = (G_1 + G_2)X$$
$$\frac{Y(s)}{X(s)} = \frac{G_1 + G_2}{1 + H(G_1 + G_2)}$$

151. System is second order

It means, number of poles = 2

System has perfectly flat magnitude response.

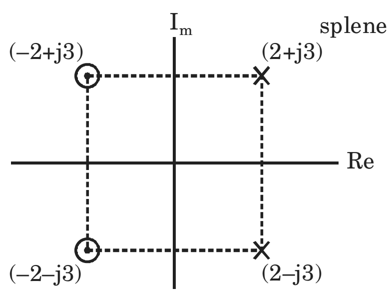
It means, It is all pass system

In all pass system,

Poles and zeros are mirror image about Imaginary axis,

One pole = $(2 - j3)$

Other pole = $(2 + j3)$



Therefore

poles at $(2 \pm j3)$

zeros at $(-2 \pm j3)$

152. Open loop transfer function of a third order unity feedback system

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

Now zero is introduce at –3.

Modified transfer function,

$$G(s) = \frac{K(s+3)}{s(s+1)(s+2)}$$

Closed loop characteristic equation,

$$1 + G(s) = 0$$

$$1 + \frac{K(s+3)}{s(s+1)(s+2)} = 0$$
$$s(s + 1)(s + 2) + K(s + 3) = 0$$
$$s^3 + 3s^2 + 2s + Ks + 3K = 0$$
$$s^3 + 3s^2 + (K + 2)s + 3K = 0$$

Routh array corresponding to characteristic equation,

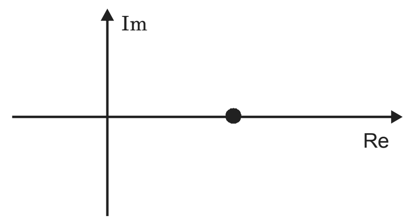
s^3	1	$(K+2)$
s^2	3	3K
s^1	2	0
s^0	3K	0

For every value of K > 0

Ist column of the array are positive.

Therefore, Root locus of modified system never transits to unstable region.

153.



$$G_1(s) = \frac{1}{s} \text{ and } G_2(s) = s$$

From the Nquist plot,

$$\therefore \quad G_1G_2(s) = \frac{1}{s} \times s = 1$$

154. The characteristic equation of given function is

$$1 + G(s)\, H(s) = 0$$

$\therefore$ $s(s^2 + 3s + 2) + K = 0$ $\quad (\because H(s) = 1)$

$\therefore$ $-k = s^3 + 3s^2 + 2s$

In order to find break away point

We have $\frac{dK}{ds} = 0$

$\therefore$ $3s^2 + 6s + 2 = 0$

$\therefore$ S = –0.42 is the solution that makes $k > 0$

155. Here $A = \left|\frac{j\omega}{j\omega+2}\right|_{\omega=2} = \frac{2}{\sqrt{2^2+2^2}} = \frac{2}{2\sqrt{2}} = \frac{1}{\sqrt{2}}$

$$\phi = \angle\frac{j\omega}{j\omega+2}\bigg|_{\omega=2}$$
$$= 90° - \tan^{-1}\frac{2}{2}$$
$$= 90° - \tan^{-1}(1)$$
$$= (90° - 45°) = 45°$$

So, the value of A and ϕ are $\frac{1}{\sqrt{2}}$, +45° respectively

156. Given transfer function $G(s) = \frac{100}{(s+1)^3}$

$$\therefore \quad \angle\frac{100}{(j\omega+1)^3} = -180^\circ\Big|_{\omega=\omega_{pc}}$$

$$-3\tan^{-1}\omega_{pc} = -180^\circ$$

$$\therefore \quad \omega_{pc} = \sqrt{3}$$

So, the phase cross-over frequency of the given transfer function is $\sqrt{3}$.

157. From the given Bode plot the corner frequencies are 2 rad/sec and 4 rad/sec respectively.

$$\text{Transfer function (TF)} = \frac{Ks}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{4}\right)^2}$$

$20\log K + 20\log\omega = 0$ dB

At $\omega = 0.5$, we get

$$20\log K + 20\log 0.5 = 0$$

$$20\log K = -20\log\left(\frac{1}{2}\right)$$

$$20\log K = 20\log(2)$$

$$\therefore \quad K = 2$$

$$\therefore \quad TF = \frac{2s}{(1+0.55)(1+0.25s)^2}$$

158. Given transfer function $G(s).H(s) = \frac{s+3}{s^2(s-3)}$

$$CE = 1 + G(s).H(s) = 1+\frac{s+3}{s^2-3s^2} = 0$$

$$s^3 - 3s^2 + s + 3 = 0$$

S^3	1	1
S^2	–3	3
S^1	2	
S^0	3	

System is unstable with two right half of s-plane poles

$\therefore$ Z = 2, P = 1

N = P – Z

N = 1 – 2 = –1 (it shows once in clockwise direction.

159. Let
$$I = 2\int_{-\infty}^{\infty}\left(\frac{\sin 2\pi t}{\pi t}\right)dt \quad ...(i)$$

As per the fourier transform of

$$\frac{2\sin(t\tau/2)}{t} \longrightarrow 2\pi\,\text{rect}\left(\frac{\omega}{t}\right)$$

$$\frac{\sin(2\pi t)}{\pi t} \longrightarrow \text{rect}\left(\frac{\omega}{4\pi}\right)$$

So,
$$\int_{-\infty}^{\infty}\frac{\sin(2\pi t)}{\pi t}e^{-j\omega t}dt = \text{rect}\left(\frac{\omega}{4\pi}\right) \quad ...(ii)$$

Putting $\omega = 0$ in above equation, we get

$$\int_{-\infty}^{\infty}\frac{\sin(2\pi t)}{\pi t}dt = 1$$

Now from equation (*i*)

$$\therefore \quad I = 2\int_{-\infty}^{\infty}\frac{\sin(2\pi)}{\pi t}dt = 2\times 1 = 2$$

160. Given
$$P = \begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$$

According to question

$$P\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}a\\ b\end{bmatrix}$$

$$\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}a\\ b\end{bmatrix}$$

$$3x + y = a$$

$$x + 3y = b$$

$$a^2 + b^2 = 1$$

$\Rightarrow \quad 10x^2 + 10y^2 + 12xy = 1$

Ellipse with major axis along $\begin{bmatrix}1\\ 1\end{bmatrix}$.

161. Through open loop transfer function

$$G(s) = \frac{K(s+1)}{s(1+Ts)(1+2s)}, K > 0, \text{ and } T > 0$$

For closed loop system stability, characteristic equation is

$$1 + G(s)H(s) = 0$$

$$1 + \frac{K(s+1)}{s(1+Ts)(1+2s)}\cdot 1 = 0$$

$s(1 + Ts)(1 + 2s) + k(s + 1) = 0$

$2Ts^3 + (2 + T)s^2 + (1 + k)s + k = 0$

Using Routh's criteria

s^3	$2T$	$(1+k)$
s^2	$(2+T)$	k
s^1	$\frac{(2+T)(1+k)-2Tk}{(2+T)}$	0
s^0	k	

For stability, $\quad k > 0$

and $\quad (2 + T)(1 + k) - 2Tk > 0$

$$k(2 + T - 2T) + (2 + T) > 0$$

or $\quad -(T - 2)k + 2(2 + T) > 0$

$$-k > -\frac{(2+T)}{(T-2)}$$

or $$k < \frac{T+2}{(T-2)}$$

Hence the closed loop system will be stable, if

$$0 < k < \frac{T+2}{T-2}$$

162. Damping ratio

$$\xi = 0.5$$

Undamped natural frequency

$$\omega_n = 10 \text{ rad/sec}$$

Steady state output to a unit step input

$$C_{ss} = 1.02$$

Hence steady state error = 1.02 – 1.00

$$e_{ss} = 0.02$$

∵ Required characteristics equation is,

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

$$s^2 + 2\times 0.5\times 10s + 100 = 0$$

$$s^2 + 10s + 100 = 0$$

From options, if we take option (*b*)

then $$C_{ss} = \lim_{s\to 0} s.C(s)$$

$$= \lim_{s\to 0} s\times\frac{1}{s}\times\frac{102}{s^2+10s+100}$$

$$C_{ss} = 1.02$$

Hence the transfer function of a system is $\frac{1.02}{s^2+10s+100}$.

163. Given, the gain at breakaway point

Open loop transfer function

OLTF ⇒ $$G(s) = \frac{Ks}{(s-1)(s-4)}$$

Now, characteristics equation is

$$1 + G(s)H(s) = 0$$

$$\frac{Ks}{(s-1)(s-4)} + 1 = 0$$

⇒ $$Ks + (s^2 + 5s + 4) = 0$$

$$K = -\frac{(s^2-5s+4)}{s} = \left[s - 5 + \frac{4}{s}\right]$$

For break away point :

$$-\frac{dK}{ds} = 0$$

$$-\frac{dK}{ds} = -\left[1 - 0 - \frac{4}{s^2}\right] = 0$$

we get $s = \pm 2$

Therefore valid break away point is

$$s = 2,$$

Now gain at $s = 2$ is

⇒ $$K = \frac{\text{Product of distances from all the poles to break away point}}{\text{Product of distance from all the zeros to break away point}}$$

Gain, $$K = \frac{1\times 2}{2} = 1$$

Hence the value of K is 1.

164. The characteristic equation is given by

$$s^3 + Ks^2 + (K+2)s + 3 = 0$$

For this system to be stable, using Routh's criterion, we get

$$3 < K(K+2)$$

$K^2 + 2K - 3 > 0$

$K^2 + 3K - K - 3 > 0$

$K(K+3) - 1(K+3) > 0$

$(K+3)(K-1) > 0$

So from the given option, $K > 1$ conditions should be satisfied

165. The transfer function of a system is given by

$$\frac{V_0(s)}{V_1(s)} = \frac{1-s}{1+s} = H(s)$$

For the minimum and maximum values of "ϕ"

$$H(j\omega) = \frac{1-j\omega}{1+j\omega}$$

∴ $$\angle H(j\omega) = -2\tan^{-1}\omega$$

At $\omega = 0$; $\angle H(j\omega) = 0°$

At $\omega = \infty$; $\angle H(j\omega) = -\pi$

Therefore the minimum and maximum value of ϕ are $-\pi$ and 0.

166. Given, $$\dot{X} = \begin{bmatrix}1 & 2\\ 2 & 0\end{bmatrix}X + \begin{bmatrix}1\\ 2\end{bmatrix}u$$

and $$Y = \begin{bmatrix}1 & 0\end{bmatrix}X$$

Transfer function = $C[sI - A]^{-1}B + D$

$$[sI - A]^{-1} = \begin{bmatrix}\frac{s}{s^2-s-4} & \frac{2}{s^2-s-4}\\ \frac{2}{s^2-s-4} & \frac{s-1}{s^2-s-4}\end{bmatrix}$$

$$[sI - A]^{-1}\cdot B = \begin{bmatrix}\frac{s+4}{s^2-s-4}\\ \frac{s-1}{s^2-s-4}\end{bmatrix}$$

∴ $$\frac{Y(s)}{U(s)} = C[sI-A]^{-1}\cdot B = \frac{s+4}{s^2-s-4}$$

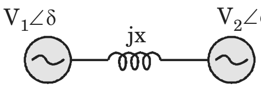
So the transfer function of the system $\frac{Y(s)}{U(s)}$ is $\frac{s+4}{s^2-s-4}$

167. From the given fig.

$$\text{Transfer function, } \left(\frac{Y(s)}{U_1(s)}\right) = \frac{\frac{K_1}{LJs^2}[1]}{1-\left[-\frac{R}{LS}-\frac{K_1K_2}{LJs^2}\right]}$$

$$= \frac{K_1}{LJs^2 + RJs + K_1K_2}$$

168.

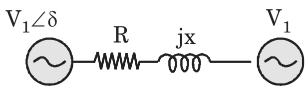


Assuming loss-less line

$$P = \frac{v_1 v_2}{X}\sin\delta$$

$$\text{Max. power} = P_{max} = \frac{v_1 v_2}{X}$$

For Lossy-Line,



Because of Resistance there is a power loss

Power Loss = $3I^2R$

$$\therefore \text{ Lossy Line } P_{max} < \frac{v_1 v_2}{X}$$

169. $$\frac{15}{s^2+5s+15} = \frac{\omega_{n_1}^2}{s^2+2\xi_1\omega_{n_1}s+\omega_{n_1}^2}$$

$$\omega_{n_1} = \sqrt{15} = 3.872$$

$$2G_1\omega_{n_1} = 5$$

$$\Rightarrow \quad G_1 = \frac{5}{2\times 3.872} = 0.64 \qquad \text{under damped}$$

$$\frac{25}{s^2+10s+25} = \frac{\omega_{n_1}^2}{s^2+2\xi_2\omega_{n_2}s+\omega_{n_2}^2}$$

$$\omega_{n_2}^2 = 25 \Rightarrow \omega_{n_2} = \pm 5\text{rad/sec}$$

$$2\xi_2\omega_{n_2} = 10$$

$$\Rightarrow \quad \xi_2 = \frac{10}{2\times 5} = 1 \qquad \text{critically damped}$$

170. $s^7 + s^6 + 7s^5 + 14s^4 + 31s^3 + 73s^2 + 25s + 200 = 0$

s^7	1	7	31	25
s^6	1	14	73	200
s^5	–7	–42	–175	200
s^4	8	48	200	
s^3	4	12		→ Row of zeros
s^2	24	200		
s^1	–170			
s^0	200			

Auxiliary equation $8s^4 + 48s^2 + 200$

A.E. = $s^4 + 6s^2 + 25 = 0$

$$\frac{d(A\cdot E)}{ds} = 4s^3 + 12s$$

A.E: $s^4 + 6s^2 + 25 = 0$

$s = -3 \pm j4 \to$ 2 poles in LHS

$= +3 \pm j4 \to$ 2 poles in RHS.

2 sign changes in above A.E $\Rightarrow$ 2 poles in RHS

Total poles = 7

Poles in RHS = 2 + 2 = 4

Poles in LHS = 7 – 4 = 3

[No poles on Imaginary axis]

171. The given characteristic equation of LTI system is

$$\Delta(s) = s^4 + 3s^3 + 3s^2 + s + k \text{ , for}$$

According to Routh hurwitz criteria,

s^4	1	3	k
s^3	3	1	0
s^2	$8/3$	k	
s^1	$\frac{8/3-3k}{8/3}$		
s^0	k		

For stability all elements of 1st column should be positive.

$$\frac{\frac{8}{3}-3k}{8/3} > 0 \text{ and } k > 0$$

$$\therefore \; k < \frac{8}{9} \text{ and } k > 0$$

$$\therefore \; 0 < k < \frac{8}{9}$$

172. It is given that

$$H(s) = \frac{a_1s^2 + b_1s + c_1}{a_2s^2 + b_2s + c_2}$$

If $a_1 = b_1 = 0$, then H(s) becomes

$$H(s) = \frac{0+0+4}{a_2s^2 + b_2s + c_2}$$

$$H(s) = \frac{c_1}{a_2s^2 + b_2s + c_2}$$

Now $H(0) = \frac{c_1}{c_2}$ (i.e., as low frequency $s \to 0 \Rightarrow \omega \to 0$)

$H(\infty) = 0$(i.e., as high frequency $s \to \infty \Rightarrow \omega \to \infty$)

So the system passes low frequency and blocks high frequency. So it works as a low pass filter.

173. $G(s) = \dfrac{\pi e^{-0.25s}}{s}$,

Now when the Nyquist Plot psses through negative real axis then in phase become (–180°)

$\therefore \quad -180° = -90° - 0.25\omega \dfrac{180°}{\pi}$

$\therefore \quad \omega = 2\pi$

Now Magnitude at this frequency

$$G(2\pi) = \left|\frac{\pi e^{-0.25j(2\pi)}}{j2\pi}\right| = \frac{1}{2} = 0.5$$

At negative real axis the co-ordinate becomes (–0.5, j0)

174. → From the given bode-plot, we can say

→ At origin, there is a pole at origin, since the initial slope is – 20 db/dec.

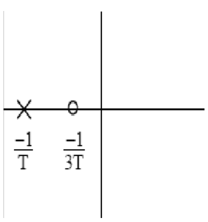
→ At ω = 20, the change in slope is – 60 – (– 40) = – 20 db/dec, so it imply one pole at ω = 20.

→ At ω = 1, the change in slope is – 40 – (– 20) = –20 dB/sec, so it imply one pole at ω = 1.

→ So in total the transfer function has 3 poles, hence at ω = ∞, the net phase contributed by 3 poles is – 270°

→ Hence statement I is false and II is true.

175.



The transfer function of a lead compensator is

$$D(s) = 3\left[\frac{s + \frac{1}{3T}}{s + \frac{1}{T}}\right]$$

The frequency at which phase is maximum is given by geometric mean of pole and zero location,

$$\omega_m = \sqrt{\left(\frac{1}{3T}\right)\left(\frac{1}{T}\right)} = \sqrt{\frac{1}{3T^2}}$$

176. From the given state space model, we can say that

$$A = \begin{bmatrix} 0 & 1 \\ -\alpha & -2\beta \end{bmatrix},\ B = \begin{bmatrix} 0 \\ \alpha \end{bmatrix},\ C = [1 \quad 0]$$

Transfer function

Now $T(s) = C(sI - A)^{-1}B$

$$= [1 \quad 0]\left[\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -\alpha & -2\beta \end{pmatrix}\right]^{-1}\begin{bmatrix} 0 \\ \alpha \end{bmatrix}$$

$$= [1 \quad 0]\begin{bmatrix} s & -1 \\ \alpha & s-2\beta \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ \alpha \end{bmatrix}$$

$$= [1 \quad 0]\left[\frac{1}{s(s+2\beta)+\alpha}\begin{bmatrix} s+2\beta & 1 \\ -\alpha & s \end{bmatrix}\right]\begin{bmatrix} 0 \\ \alpha \end{bmatrix}$$

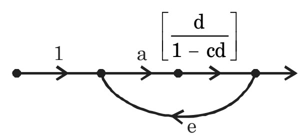
$$= [1 \quad 0]\left[\frac{1}{s^2+2\beta s+\alpha}\right]\begin{bmatrix} \alpha \\ \alpha s \end{bmatrix} = \frac{[1 \quad 0]\begin{bmatrix} \alpha \\ \alpha s \end{bmatrix}}{s^2+2\beta+\alpha}$$

$= \dfrac{\alpha}{s^2+2\beta s+\alpha}$ by comparing with standard equation $+\,\omega_n^2$ we get

$\omega_n = \sqrt{\alpha}$ and $\xi\omega_n = \beta$

$\therefore \quad \xi = \dfrac{\beta}{\sqrt{\alpha}}$

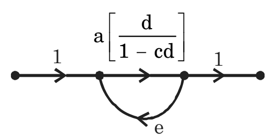
177. Apply the Mason's gain formula to the given, SFG T.F $= \dfrac{ad}{1 - ade - dc}$ ...(1)



apply the Mason's gain formula to the optionn (c)

$$\text{T.F} = \frac{a\dfrac{d}{1-cd}}{1 - \dfrac{ade}{1-cd}} = \frac{ad}{1 - ade - dc} \quad ...(2)$$

(1) = (2)



178. Given that the final value to a step of 5 V is 10 and T = 0.5s

$\dfrac{C(s)}{R(s)} = \dfrac{K}{1 + TS}$

$\therefore \quad \text{T.F} = \dfrac{2}{1 + 0.5s}$

Final value $\underset{s\to 0}{\text{Lt}} S(\text{TF})\dfrac{5}{S} = S\left(\dfrac{2}{1+0.5s}\right)\cdot\dfrac{5}{S} = 10$

$\text{T.F} = \dfrac{2}{1 + 0.5s}$

179. Characteristic Equation = 1 + G(s) H(s) = 0

$\therefore \quad 1 + \dfrac{1}{(s+1)}\dfrac{1}{s^2}\dfrac{20}{(s+20)} = 0$

$(s^3 + s^2)(s + 20) + 20 = 0$

$s^4 + 21s^3 + 20s^2 + 20 = 0$

The given system is of 4^{th} order

$+s^4$	1	20	20
$+s^3$	21	0	0
$+s^2$	20	20	0
$-s^1$	-21		
$+s^0$	20		

Number of sign changes in the first column = 2

$\therefore$ two roots are present in the right half of s-plane, and system is unstable.

180. Given transfer function $= \dfrac{s^2+s+1}{s^3+2s^2+2s+k} = 0$

Now, characteristic equation is given by

$$1 + G(s) + H(s) = 0$$

$\therefore\ s^3 + 3s^2 + 3s + (k+1) = 0$

Now according to Routh Hurwitz Criteria,

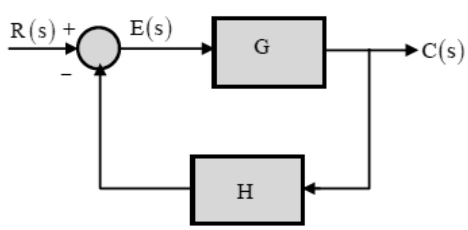
s^3	1	3
s^2	3	k + 1
s^1	$\dfrac{9-(k+1)}{3}$	
s^0	k + 1	

For marginal stability,

$\therefore\ 9-(k+1) = 0$

$\therefore\ k = 8$

181. As given circuit:



So, $\dfrac{C(s)}{R(s)} = \dfrac{G}{1+GH}$ (standard negative feedback system)

$E(s) = R(s) - C(s)H$

$$= R(s) - \frac{GH}{1+GH}R(s) = R(s)\left[1-\frac{GH}{1+GH}\right]$$

$$= R(s)\left[\frac{1+GH-GH}{1+GH}\right] = R(s)\frac{1}{1+GH}$$

$$\Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1+GH}$$

Hence, option (c) is the correct answer.

182. As given function;

$$H(s) = \frac{As+B}{s^2+Cs+D}$$

At low = freq. (s = 0)

$$H(0) = \frac{B}{D} \text{ (exist)}$$

At high – freq. ($s = \infty$)

$H(\infty) = 0$ (No output)

Since, $H(\infty)$ should be non-zero for HPF. So, the above system cannot be operated as HPF because for high pass filter should pass high-frequency component i.e $H(\infty)$ should be non-zero.

Hence, the system cannot operate as high pass filter.

183. For, the first order system, there exist one finite pole, by observing magnitude of plot, it is maintaining constant magnitude, this exist for all pass system.

So, Possibilities, $\quad T.F = \dfrac{s-1}{s+1}$ or $\dfrac{1-s}{1+s}$

Observe phase angle at $w \to \infty$

$\phi = -180°$

Number of poles, P = 1

Number of zeros, Z = 1

It is having one pole at left and one zero at right.

184. As we know that

Characteristics equation, $1 + G(s) = 0$

So, $\quad 1 + \dfrac{K}{s^2+2s-5} = 0$

Now, $s^2 + 2s + (K-5) = 0$

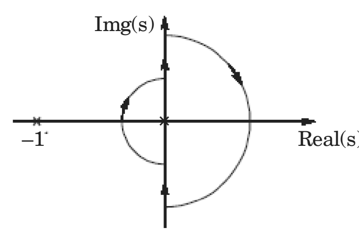
By Routh table analysis,

$K - 5 > 0,\ K > 5$ and $K > 0$

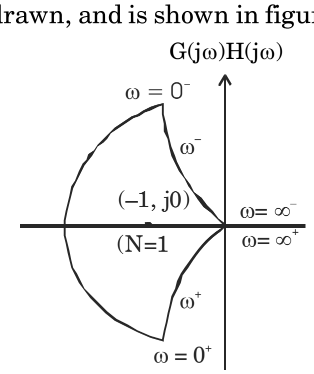
For stability $K > 5$

185. In given figure;

Given Nyquist contour S–plane



Corresponding to the Nyquist contour, the Nyquist plot is drawn, and is shown in figure below,



From the plot

Number of Encirclement (N) = 1

186. Transfer function of given system is

$$T.F = \frac{C(s)}{R(s)} = \frac{100}{s^2 + 10s + 100}$$

By compair it to standard transfer function

$$T.F = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

$$2\xi\omega_n = 10$$

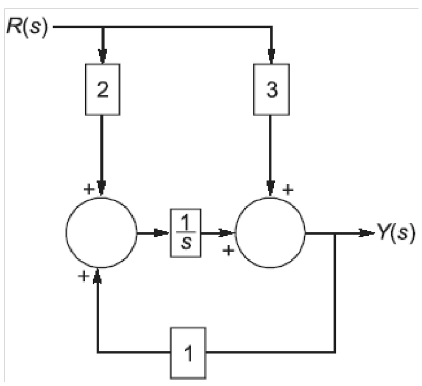
So, $\omega_n^2 = 100$

$\Rightarrow$ $\omega_n = 10$ rad/sec

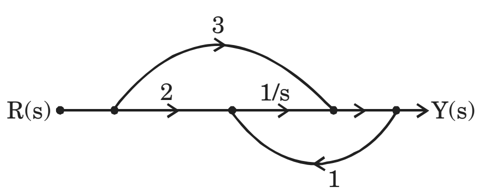
$$\xi = \frac{10}{2 \times 10}$$

$$\xi = 0.5$$

187. As given figure :



From signal flow graph;

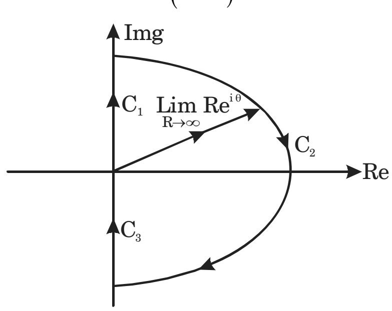


Now by using Masson's graph formula.

$$\frac{Y(s)}{R(s)} = \left[\frac{P_1\Delta_1 + P_1\Delta_2}{(1 - L_1)}\right] = \frac{3 + \frac{2}{s}}{1 - \left(\frac{1}{s}\right)} = \frac{3s + 2}{(s - 1)}$$

188. As given transfer function;

$$GH = \frac{3s + 5}{(s - 1)}$$



Now for mapping C_2,

$$G(s)\,H(s) = \lim_{R\to\infty}\left[\frac{3\,Re^{j\theta} + 5}{Re^{j\theta} - 1}\right] \qquad (\therefore S = Re^{j\theta})$$

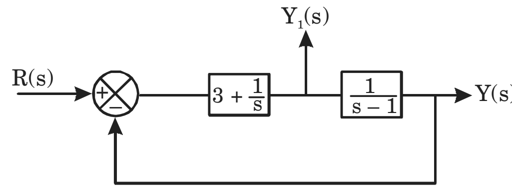
So, $\boxed{G(s)\,H(s) = 3}$

189. As given that,

$$G(s) = \frac{1}{s - 1}$$

$$H(s) = 1$$

According to the question



$$\text{Open loop transfer function} = \left[\frac{(3s + 1)}{s(s - 1)}\right]$$

$$\text{Closed loop transfer function} = \left[\frac{Y(s)}{R(s)}\right] = \left[\frac{3s + 1}{s^2 + 2s + 1}\right]$$

Final value of plant :

$$Y_{ss} = \lim_{s\to 0} sY(s)$$

$$Y_{ss} = \lim_{s\to 0} s \times \left[\frac{3s + 1}{s^2 + 2s + 1}\right] \times \frac{1}{s} = +1 \qquad \left[\therefore R(s) = \frac{1}{s}\right]$$

As now, $$Y(s) = Y_1(s) \times \left[\frac{1}{s - 1}\right]$$

$$Y_1(s) = (s - 1)\,Y(s)$$

$$Y_1(s) = (s - 1) \times \left[\frac{3s + 1}{s(s^2 + 2s + 1)}\right]$$

$$(Y_1)_{ss} = \lim_{s\to 0} sY_1(s) = -1$$

190. As given transfer fucntion;

$$K(s) = \frac{1 + \frac{s}{\alpha}}{1 + \frac{s}{\beta\alpha}}, \alpha > 0,\ \beta > 1$$

$$K(s) = \frac{s + \alpha}{(s + \beta\alpha)}$$

Also given that, the maximum phase lead occur at,

$$(\omega_m) = \sqrt{\alpha(\alpha\beta)} = 4 \text{ rad/sec}$$

$$\alpha\sqrt{\beta} = 4 \qquad \text{...(i)}$$

As we know that

$$\text{Amplification, } M = -20\log_{10}\left[\frac{\alpha\sqrt{\beta}}{\alpha}\right]$$

$$6 = 20\log_{10}\left[\sqrt{\beta}\right] \quad \begin{pmatrix}\therefore \text{From (i)} \\ \therefore M = 6\end{pmatrix}$$

$$6 = 20\log_{10}\left[\frac{4}{\alpha}\right]$$

So, $\alpha = 2$

Now, from eq. (i);

$$2\sqrt{\beta} = 4$$

$\Rightarrow$ $\beta = 4$

NUMERICAL TYPE QUESTIONS

1. The characteristic equation is given by

$$1 + GH = 0$$

or $1 + \frac{(s-5)}{s+4} K = 0$

or $s + 4 + (s - 5) K = 0$

or $s(K + 1) + 4 - 5K = 0$

This single root is given by

$$s = \frac{5K-4}{K+1} \le 0 \text{ for stability, since } K \ge 0$$

or $5K - 4 \le 0$, since $K \ge 0$,

or $K \le \frac{4}{5}$ for stability

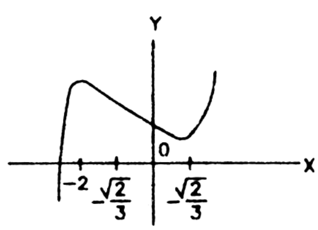
2. True

3. *One*

$$f(s) \equiv s^3 - 2s + 2 = 0$$

$$\left.\begin{aligned} f(-2) &= -8 + 4 + 2 = -ve \\ f(-1) &= -1 + 2 + 2 = +ve \end{aligned}\right]$$

There is one real root between –2 and –1



$f'(s) = 3s^2 - 2$ and $f'(s) = 0 \Rightarrow s = \pm\sqrt{\frac{2}{3}}$

$f''(s) = 6s = -\text{ve}$ when $s = -\sqrt{\frac{2}{3}}$ $= +\text{ve}$

when $s = \sqrt{\frac{2}{3}}$

$\therefore$ $f(s)$ is maximum when $s = -\sqrt{\frac{2}{3}}$

and minimum when $s = \sqrt{\frac{2}{3}}$

Maximum value of $f(s) = 2 + \sqrt{\frac{32}{27}}$

Minimum value of $f(s) = 2 - \sqrt{\frac{32}{27}} = +\text{ve}$

Graph cuts the X–axis at one point only. No other real root.

4. Roots

5. From the characteristic equation

$$1 + G(s) H(s) = 0$$

$$1 + \frac{(s+\alpha)}{s^3 + (1+\alpha)s^2 + (\alpha-1)s + (1-\alpha)} = 0$$

$\therefore$ $s^3 + (1 + \alpha)s^2 + \alpha s + 1 = 0$

By Routh Hurwitz criteria

$(1 + \alpha)\,\alpha > 1$

$(\alpha^2 + \alpha - 1) > 0$

$\therefore$ $\alpha = 0.618$ & -0.618

But for system to be stable

$\alpha = 0.618$

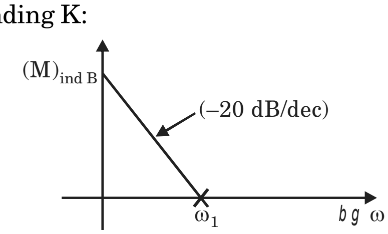
6. From the magnitude plot,

$$G(s) = \frac{K\left[1+\frac{S}{2}\right]\left[1+\frac{S}{4}\right]}{S\left[1+\frac{S}{8}\right]\left[1+\frac{S}{24}\right]\left[1+\frac{S}{36}\right]}$$

Now comparing with given transfer function

$$a = \frac{1}{4};\ b = \frac{1}{24}$$

For finding K:



$K = (\omega)n$: where n is no. of poles from the given plot

$\therefore$ $K = (8)^1$

Now , $\frac{a}{bk} = \left(\frac{1}{24}\times 8\right) = 0.75$

7. $T(s) = \frac{C(s)}{R(s)} = \frac{4}{s^2 + 0.4s + 4}$

$$\Rightarrow \frac{R(s) - C(s)}{R(s)} = \frac{E(s)}{R(s)} = \frac{s^2 + 0.4s}{s^2 + 0.4s + 4}$$

Steady- state error

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0}\frac{sR(s)(s^2+0.4s)}{s^2+0.4s+4}$$

Here, $R(s) = \frac{1}{s}$

Then, $e_{ss} = \lim_{s\to 0}\frac{s^2+0.4s}{s^2+0.4s+4} = 0$

8. Characteristic equation is, $1 + G(s)H(s) = 0$

$\Rightarrow \quad 1 + G(s) = 0 \qquad (\because H(s) = 1)$

$$\Rightarrow 1+\frac{k}{s(s+2)(s^2+2s+2)} = 0$$

$\Rightarrow s^4 + 4s^3 + 6s^2 + 4s + k = 0$

Constructing Routh- array, we have

s^4	1	6	k
s^3	4	4	0
s^2	5	k	
s^1	$\frac{20-4k}{5}$	0	

For the closed loop system to be marginally stable, $20 - 4k = 0$

$\Rightarrow \quad k = 5$

9. $g(t) = e^{-2}[\sin 5t + \cos 5t]$

Taking lapcase transform of $g(t)$,

$$\therefore \quad G(s) = \frac{5}{(s+2)^2+5^2}+\frac{s+2}{\{s+2\}^2+5^2}$$

For DC gain, $|G(s)|_{s=0}$

$$G(0) = \frac{5}{2^2+5^2}+\frac{2}{2^2+5^2} = \frac{7}{29}$$

10. $H(s) = \frac{1}{(s+1)}$

Put $s = j\omega$, $H(j\omega) = \frac{1}{j\omega+1}$

$$|H(j\omega)| = \frac{1}{\sqrt{\omega^2+1}}$$

$\because$ input $x(t) = \cos(t)$

Here $\omega = 1$ rad/sec

and $|x(t)| = 1$

Hence, steady state output

$$y(t) = |x(t)| \times |H(j\omega)|_{\omega=1} \cos\left[t + \angle H(j\omega)\right]$$

$$A = |x(t)| \times |H(j\omega)|_{\omega=1}$$

$$A = \frac{1}{\sqrt{2}} = 0.707$$

So, the value of A is 0.707.

11. $E = \int_C \bar{F}\cdot \bar{dr}$

Here $\bar{F} = 5xz\hat{i} + \left(3x^2+2y\right) - \hat{j} + x^2 2\hat{k}$

$$= \int_C 5xzdx + \left(3x^2+2y\right)dy + x^2zdz$$

Put $x = t$,

$y = t^2$,

$z = t$,

$t = 0$ to 1

$dx = dt$

$dy = 2t\ dy$,

$dz = dt$

$$= \int_0^1 5t^2dt + \left(3t^2+2t^2\right)2tdt + t^3dt$$

$$= \int_0^1 \left(5t^2 + 11t^3\right)dt$$

$$= \left[\frac{5t^3}{3}+\frac{11t^4}{4}\right]_0^1$$

$$= \frac{5}{3}+\frac{11}{4}$$

$$= \frac{53}{12} = 4.41$$

12. From the given fig.

Forward path transfer function,

$$G(s) = \frac{Ke^{-s}}{s}$$

Given, Phase margin = 30°

or Phase margin = 180° + ϕ

$30° = 180° + \phi$

$\therefore \quad \phi = -150°$

Now $\phi = \angle G(j\omega)_{\omega=\omega_{gc}}$

[where, ω_{gc} = gain crossover frequency]

$\angle G(j\omega) = -90° - 57.3\,\omega°$

At $\omega = \omega_{gc}$

[gain crossover frequency]

$|G(j\omega)| = 1$

or $\frac{K\times 1}{\omega} = 1 \Rightarrow \omega = \omega_{gc} = K$ rad/sec.

$\therefore \quad \angle G(j\omega)_{\omega=\omega_{gc}} = -90° - 57.3\,K$

$-90° - 57.3\,K = -150°$

$-57.3\,K = -60°$

$$\therefore \quad K = \frac{60}{57.3} = 1.047$$

Hence the value of K is 1.047

13.
$$G(s) = \frac{-s+1}{s+1}$$

System output,

$$Y(s) = G(s)\cdot\frac{1}{s}$$
$$= \frac{-s+1}{s+1}\cdot\frac{1}{s}$$
$$= \frac{1}{s} - \frac{2}{s+1}$$
$$y(t) = u(t) - 2e^{-t}\,u(t)$$
$$\therefore \quad y(1.5) = 1 - 2e^{-1.5}$$
$$= 1 - 0.446 = 0.554$$

So the value of the response of the system at t =1.55 e is 0.554.

14. $G(s) = \frac{1}{(s+1)(s+2)}$

$H(s) = 1$

Steady state error due to unit step input is

$$e_{ss} = \frac{A}{1 + \underset{s\to 0}{Lt}\, G(s)}$$
$$= \frac{1}{1 + \underset{s\to 0}{Lt}\frac{1}{(s+1)(s+2)}} = \frac{1}{1 + \frac{1}{(0+1)(0+2)}}$$
$$= \frac{1}{1+\frac{1}{2}} = \frac{2}{3} = 0.66$$

15. Steady-state error = e_{ss} = 1–0.8 = $\frac{1}{1+K_p}$

$$1 + K_p = \frac{1}{0.2} = 5 \Rightarrow K_p = 4$$
$$K_p = \underset{s\to 0}{Lt}\, G(s) = \underset{s\to 0}{Lt}\frac{K}{(s+1)^2(s+2)}$$
$$4 = \frac{K}{(0+1)^2(0+2)} \Rightarrow K = 8 = 4\times 2$$

16. As given that

$$O.L.T.F = G(s) = \frac{K}{(s+a)(s-b)(s+c)}$$

$P = 1$

$N = -1$

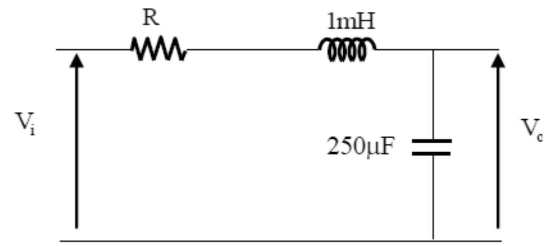
$N = P - Z$

$Z = P - N = 1 - (-1)$

$Z = 2$

Hence, $\frac{G(s)}{1+G(s)}$ has two poles in the right half of s-plane.

17. For the following RLC circuit, the gain at frequency 2000 rad/sec is given 26dB. We need to obtain value of R.



When the operating frequency is 2000 rad/sec

$Z_L = jwL = j \times 2000 \times 1 \times 10^{-3} = j2$

$$Z_c = \frac{-j}{\omega C} = \frac{-1}{200\times 250\times 10^{-6}} = -j2$$

In general

$$V_o(j\omega) = \frac{Z_C}{R + Z_L + Z_C}V_i(j\omega)$$
$$\Rightarrow \frac{V_o(j\omega)}{V_i(j\omega)} = \frac{-j2}{R + j2 - j2}(\text{at } \omega = 2000\text{r/sec})$$
$$\Rightarrow \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = \frac{2}{R} \quad \text{...(i)}$$

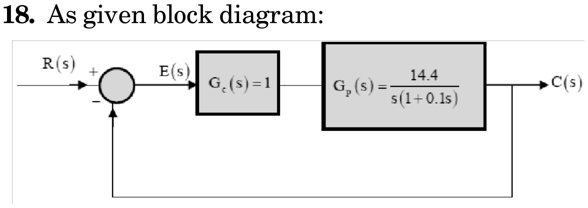
As given that,

$$20\log\left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = 26$$
$$\Rightarrow \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = 10^{\frac{26}{20}} \ (\text{at } \omega = 2000 \text{ rad/sec})$$
$$\Rightarrow \left|\frac{V_o(j\omega)}{V_i(j\omega)}\right| = 19.95 \quad \text{...(ii)}$$

Equating equation (i) and (ii) $\frac{2}{R} = 19.95$

$$\Rightarrow R = \frac{2}{19.95} \qquad \Rightarrow R = 0.1\Omega$$

18. As given block diagram:



As given that,

For unit step input response has damped oscillation we need to obtain damped natural frequency.

Now, we get,

The closed loop transfer function for the given diagram.

$$\frac{C(s)}{R(s)} = \frac{G(s)G(s)}{1+G_c(s)Gp(s)} = \frac{\frac{14.4}{s(1+0.1s)}}{1+\frac{14.4}{s(1+0.1s)}}$$

$$= \frac{14.4}{s(1+0.1s)+14.4} = \frac{14.4}{s+0.1s^2+14.4}$$

$$= \frac{144}{s^2+10s+144} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_0^2}$$

Now, by comparison $\omega_n^2 = 144$

$\Rightarrow$ $\omega_n = 12$ rad/sec

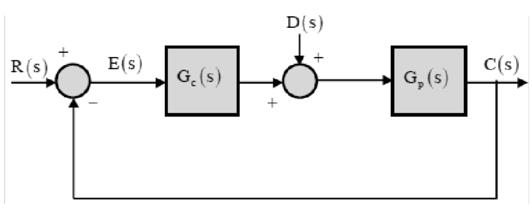
$2\xi\omega_n = 10$

$$\Rightarrow \xi = \frac{10}{2\omega_n} = \frac{10}{24} = 0.416$$

So, damped natural frequency :

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 12\sqrt{1-(0.416)^2} = 10.90 \text{rad/sec}$$

19. According to the given block diagram :



$$G_p(s) = \frac{2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}, G_c(s) = k\left(\frac{1+T_1s}{1+T_2s}\right)$$

When D(s) is unit step the steady state error should be at maximum 0.1, we need to obtain minimum value of k.

Since there are 2 inputs (R(s), D(s)), we want E(s) due to D(s), so R(s) should be made 0

We know that,

$$\frac{E(s)}{D(s)} = \frac{G_p(s)}{1+G_pG_c(s)}$$

$$\text{Now, } E(s) = \frac{-G_p(s)}{s(1+G_p(s)G_0(s))} \qquad \left(\because D(s) = \frac{1}{s}\right)$$

Steady state error $e(\infty)$ =

$$\lim_{s\to 0} sE(s) = \lim_{s\to 0}\left[\frac{-G_p(s)}{1+G_p(s)G_c(s)}\right]$$

$$\Rightarrow e(\infty) = \lim_{s\to 0}\left[\frac{\frac{-2.2}{(1+0.1s)(1+0.4s)(1+1.2s)}}{1+k\left(\frac{1+T_1s}{1+T_2s}\right)\frac{2.2}{(1+0.1s)(1+0.4s1+1.2s)}}\right]$$

$$= \frac{+2.2}{1+2.2k}$$

Now, We need $e(\infty) \le 0.1$

$$\text{So, } \frac{+2.2}{1+2.2k} \le 0.1$$

$\Rightarrow$ $k \geq 9.54$

$\Rightarrow$ $k_{min} = 9.54$

Hence, the Minimum value of k is 9.54.

20. As given that

$\dot{x} = -x+u$

$y = x$

$u = -kx$

According to the question we need to obtain value of such that closed loop pole will be at s = –2.

Closed loop pole is roots of characteristic equation,

Characteristic equation in terms of state space is

$|sI - A| = 0$

$\dot{x} = -x+u = -x-kx \qquad (\because u = -kx)$

$\Rightarrow$ $sX(s) = -X(s) - kX(s)$

$\Rightarrow$ $\dot{x} = x(-k-1)$

$\Rightarrow$ $X(s)\ [s + 1 - k] = 1$

$\Rightarrow$ $\dot{x} = Ax + BU$

By comparison A = – k – 1

As we know that,

Characteristic equation : $|sI - A| = 0$

$|s - (-k - 1) = 0|$

$|s + k + 1| = 0$

$s + k + 1 = 0$

(Determinant of a constant is constant)

$k + 1 = 2$

(closed loop pole is k + 1 and if should be at 2)

$\boxed{k = 1}$

■■