CBSE X 2025

Chapter and Topic-Wise

Solved Papers

2011-2024

Mathematics

Standard & Basic

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Career Launcher

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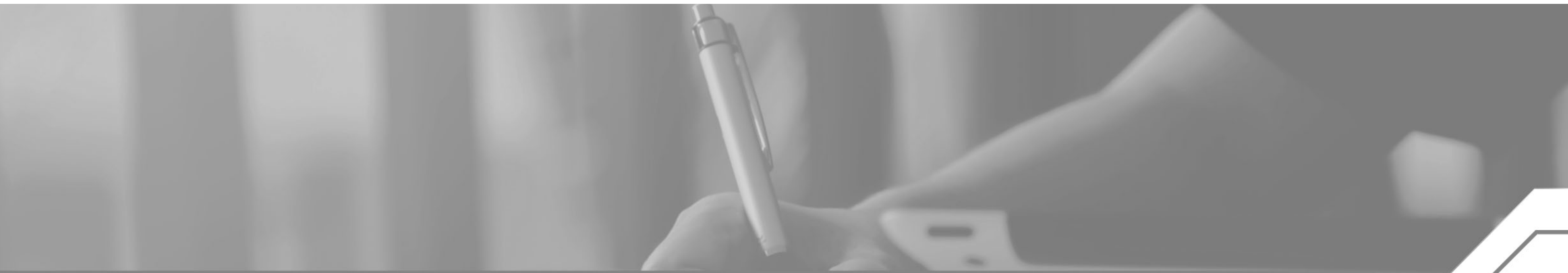

PREFACE

Class X Board Exams are a race against time. You must know how to manage time efficiently if you want to ace your exams. At Career Launcher, we understand the struggle of attempting such a crucial examination for the first time and the pressure that comes along with it. Which is why, our Chapter and Topic-Wise Solved Papers for Mathematics have been designed to help you become acquainted with the exam pattern and hone your time management skills, both at the same time.

Exclusively designed for the students of CBSE Class X by highly experienced teachers, the book provides answers to all actual questions of Mathematics Board Exams conducted from 2011 to 2024. The solutions have been prepared exactly in coherence with the latest marking pattern; after a careful evaluation of previous year trends of the questions asked in Class X Boards and actual solutions provided by CBSE.

The book follows a three-pronged approach to make your study more focused. The questions are arranged Chapter-wise so that you can begin your preparation with the areas that demand more attention. These are further segmented topic-wise and eventually the break-down is as per the marking scheme. This division will equip you with the ability to gauge which questions require more emphasis and answer accordingly. Apart from this, several value-based questions have also been included.

We hope the book provides the right exposure to Class X students so that you not only ace your Boards but mold a better future for yourself. And as always, Career Launcher's school team is behind you with its experienced gurus to help your career take wings.

Let's face the Boards with more confidence!

Wishing you all the best,

Team CL

Blueprint & Marks Distribution

Class 10th Mathematics 2024-25 Analysis Unit Wise

Unit No.	Name	No. of Periods	Marks
I	Number Systems	15	6
II	Algebra	45	20
III	Coordinate Geometry	14	6
IV	Geometry	31	15
V	Trigonometry	33	12
VI	Mensuration	24	10
VII	Statistics & Probability	28	11
	Total		**80**
	Internal Assessment		**20**
Grand Total		**190**	**100**

UNIT I: NUMBER SYSTEMS (15) PERIODS

1. REAL NUMBERS

Fundamental Theorem of Arithmetic - statements after reviewing work done earlier and after illustrating and motivating through examples, Proofs of irrationality of $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$.

UNIT II: ALGEBRA

1. POLYNOMIALS (8) PERIODS

Zeros of a polynomial. Relationship between zeros and coefficients of quadratic polynomials.

2. PAIR OF LINEAR EQUATIONS IN TWO VARIABLES (15) PERIODS

Pair of linear equations in two variables and graphical method of their solution, consistency/ inconsistency.

Algebraic conditions for number of solutions. Solution of a pair of linear equations in two variables algebraically - by substitution, by elimination. Simple situational problems.

3. QUADRATIC EQUATIONS (15) PERIODS

Standard form of a quadratic equation $ax^2+bx+c=0$, $(a \neq 0)$. Solutions of quadratic equations (only real roots) by factorization, and by using quadratic formula. Relationship between discriminant and nature of roots.

Situational problems based on quadratic equations related to day to day activities to be incorporated.

4. ARITHMETIC PROGRESSIONS (10) PERIODS

Motivation for studying Arithmetic Progression Derivation of the n^{th} term and sum of the first n terms of A.P. and their application in solving daily life problems.

UNIT III: COORDINATE GEOMETRY (15) PERIODS

REVIEW: Concepts of coordinate geometry, graphs of linear equations. Distance formula. Section formula (internal division). Area of a triangle.

UNIT IV: GEOMETRY

1. TRIANGLES (15) PERIODS

Definitions, examples, counter examples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.
4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.

2. CIRCLES (10) PERIODS

Tangent to a circle at, point of contact

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

UNIT V: TRIGONOMETRY

1 . INTRODUCTION TO TRIGONOMETRY (10) PERIODS

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios whichever are defined at 0° and 90°. Values of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

2. TRIGONOMETRIC IDENTITIES (15) PERIODS

Proof and applications of the identity $\sin^2A + \cos^2A = 1$. Only simple identities to be given.

3. HEIGHTS AND DISTANCES: Angle of elevation, Angle of Depression. (10) PERIODS

Simple problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation / depression should be only 30°, 45°, and 60°.

UNIT VI: MENSURATION

1. AREAS RELATED TO CIRCLES (12) PERIODS

Area of sectors and segments of a circle. Problems based on areas and perimeter / circumference of the above said plane figures. (In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only.

2. SURFACE AREAS AND VOLUMES (12) PERIODS

Surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones.

UNIT VII: STATISTICS AND PROBABILITY

1. STATISTICS (18) PERIODS

Mean, median and mode of grouped data (bimodal situation to be avoided).

2. PROBABILITY (10) PERIODS

Classical definition of probability. Simple problems on finding the probability of an event.

MATHEMATICS-STANDARD QUESTION PAPER DESIGN
CLASS – X (2024-25)

Time : 3 Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	43	54
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	19	24
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	18	22
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

MATHEMATICS-BASIC QUESTION PAPER DESIGN
CLASS – X (2024-25)

Time : 3Hours **Max. Marks: 80**

S. No.	Typology of Questions	Total Marks	% Weightage (approx.)
1	**Remembering:** Exhibit memory of previously learned material by recalling facts, terms, basic concepts, and answers. **Understanding:** Demonstrate understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions, and stating main ideas	60	75
2	**Applying:** Solve problems to new situations by applying acquired knowledge, facts, techniques and rules in a different way.	12	15
3	**Analysing :** Examine and break information into parts by identifying motives or causes. Make inferences and find evidence to support generalizations **Evaluating:** Present and defend opinions by making judgments about information, validity of ideas, or quality of work based on a set of criteria. **Creating:** Compile information together in a different way by combining elements in a new pattern or proposing alternative solutions	8	10
	Total	80	100

INTERNAL ASSESSMENT	**20 MARKS**
Pen Paper Test and Multiple Assessment (5+5)	10 Marks
Portfolio	05 Marks
Lab Practical (Lab activities to be done from the prescribed books)	05 Marks

CHAPTER 1

Real Numbers

[TOPIC 1] Euclid's Division Lemma and Fundamental Theorem of Arithmetic

Summary

Euclid's Division Lemma

Dividend = divisor × quotient + remainder.

Given two positive integers a and b. There exist unique integers q and r satisfying

$a = bq + r$ where $0 \leq r < b$

where a is dividend, b is divisor, q is quotient and r is remainder.

- ***If a = bq + r, then every common divisor of a and b is a common divisor of b and r also.***

Euclid's Division Algorithm

To obtain the HCF of two positive integers, say c and d, with $c > d$, follow the steps below:

Step 1: Apply Euclid's division lemma, to c and d. So, we find whole numbers, q and r such that $c = dq + r$, $0 \leq r < d$.

Step 2: If $r = 0$, d is the HCF of c and d. If $r \neq 0$, apply the division lemma to d and r.

Step 3: Write $d = er + r_1$ where $0 < r_1 < r$

Step 4: Continue the process till the remainder is zero. The divisor at this stage will be the required HCF.

- *Odd integers of the form* ***6q + 1, 6q + 3 or 6q + 5*** *shows that 6 is the divisor of given integer*
- *Any positive integer can be of the form 3m, 3m + 1, 3m + 2. Such that its cube would be of the form 9q + r.*

Fundamental Theorem of Arithmetic

Every composite number can be expressed as a product of primes and this expression is **unique**, except from the order in which the prime factors occur.

- *HCF is the lowest power of common prime and LCM is the highest power of primes.*
- *HCF (a, b) × LCM (a, b) = a × b.*
- *Any number ending with zero must have a factor of 2 and 5.*

PREVIOUS YEARS'

EXAMINATION QUESTIONS

TOPIC 1

1 Mark Questions

1. L.C.M. of $2^3 \times 3^2$ and $2^3 \times 3^3$ is:
   (a) 2^3 (b) 3^3
   (c) $2^3 \times 3^3$ (d) $2^2 \times 3^2$
   [TERM 1, 2012]

2. If p and q are two co-prime numbers, then HCF (p, q) is:
   (a) p (b) q
   (c) pq (d) 1 [TERM 1, 2013]

3. If $a = (2^2 \times 3^3 \times 5^4)$ and $b = (2^3 \times 3^2 \times 5)$, then HCF (a, b) is equal to:
   (a) 900 (b) 180
   (c) 360 (d) 540 [TERM 1, 2013]

4. The HCF of two numbers is 27 and their LCM is 162, if one of the number is 54, find the other number.
   [TERM 1, 2017]

5. What is the HCF of the smallest prime number and the smallest composite number?
   [TERM 1, 2017]

6. Write the number of zeroes in the end of a number whose prime factorization is $2^2 \times 5^3 \times 3^2 \times 17$.
   [2019]

7. The sum of exponents of prime factors in the prime-factorisation of 196 is
   (a) 3 (b) 4
   (c) 5 (d) 2 [Standard, 2020]

8. Euclid's division Lemma states that for two positive integers a and b, there exists unique integer q and r satisfying a = bq + r, and
   (a) $0 < r < b$ (b) $0 < r \leq b$
   (c) $0 \leq r < b$ (d) $0 \leq r \leq b$
   [Standard, 2020]

9. 120 can be expressed as a product of its prime factors as
   (a) $5 \times 8 \times 3$ (b) 15×2^3
   (c) $10 \times 2^2 \times 3$ (d) $5 \times 2^3 \times 3$
   [Basic, 2020]

10. The exponent of 5 in the prime factorisation of 3750 is
    (a) 3 (b) 4
    (c) 5 (d) 6
    [Standard Term 1, 2022]

11. The greatest number which when divides 1251, 9377 and 15628 leaves remainder 1, 2 and 3 respectively is
    (a) 575 (b) 450
    (c) 750 (d) 625
    [Standard Term 1, 2022]

12. If a and b are two coprime numbers, then a^3 and b^3 are
    (a) Coprime (b) Not coprime
    (c) Even (d) Odd
    [Standard Term 1, 2022]

13. If n is a natural number, then $2(5^n + 6^n)$ always ends with
    (a) 1 (b) 4
    (c) 3 (d) 2
    [Standard Term 1, 2022]

14. The LCM of two numbers is 2400. Which of the following can not be their HCF?
    (a) 300 (b) 400
    (c) 500 (d) 600
    [Standard Term 1, 2022]

15. HCF of 92 and 152 is
    (a) 4 (b) 19
    (c) 23 (d) 57
    [Basic Term 1, 2022]

16. HCF of two consecutive even numbers is
    (a) 0 (b) 1
    (c) 2 (d) 4
    [Basic Term 1, 2022]

17. The (HCF × LCM) for the numbers 50 and 20 is
    (a) 1000 (b) 50
    (c) 100 (d) 500
    [Basic Term 1, 2022]

18. For which natural number n, 6^n ends with digit zero?
    (a) 6 (b) 5
    (c) 0 (d) None
    [Basic Term 1, 2022]

19. If $p^2 = \frac{32}{50}$, then p is a/an

(a) whole number (b) integer

(c) rational number (d) irrational number

[Standard, 2023]

20. (HCF × LCM) for the number 30 and 70 is :

(a) 2100 (b) 21

(c) 210 (d) 70 [Basic, 2023]

2 Marks Questions

21. Show that 8^n cannot end with the digit zero for any natural number n. [TERM 1, 2011]

22. Euclid's algorithm, find the HCF of 240 and 228. [TERM 1, 2012]

23. Explain why $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 + 5$ is a composite number? [TERM 1, 2014]

24. Find the least positive integer which on diminishing by 5 is exactly divisible by 36 and 54. [TERM 1, 2015]

25. Express 5050 as product of its prime factors. Is it unique? [TERM 1, 2016]

26. Use Euclid's division algorithm to find the HCF of 255 and 867. [2019]

27. Check whether 6^n can end with the digit '0'(zero) for any natural number n.

OR

Find the LCM of 150 and 200 [Basic, 2020]

28. Show that 6^n can not end with digit 0 for any natural number 'n'.

OR

Find the HCF and LCM of 72 and 120.

[Standard, 2023]

3 Marks Questions

29. Show that square of any positive integer is either of the form $3m$ or $(3m+1)$ for some integer m. [TERM 1, 2011]

30. Find the LCM and HCF of 336 and 54 and verify that LCM × HCF = Product of the two numbers. [TERM 1, 2012]

31. Using Euclid's division algorithm, find whether the pair of numbers 847, 2160 are co-primes or not. [TERM 1, 2012]

32. Find HCF and LCM of 180, 252 and 324. [TERM 1, 2013]

33. Pens are sold in pack of 8 and notepads are sold in pack of 12. Find the least number of pack of each type that one should buy so that there are equal number of pens and notepads. [TERM 1, 2014]

34. Explain whether the number $3 \times 5 \times 13 \times 46 + 23$ is a prime number or a composite number. [TERM 1, 2015]

35. Find the greatest number of six digit number exactly divisible by 18, 24 and 36. [TERM 1, 2016]

36. Using division algorithm find quotient and remainder dividing $x^3 + 13x^2 + x - 2$ by $2x + 1$ [TERM 1, 2016]

37. Find HCF and LCM of 404 and 96 and verify that HCF × LCM = Product of the two given numbers. [TERM 1, 2017]

38. Find the HCF and LCM of 26, 65 and 117, using prime factorisation. [Standard, 2023]

4 Marks Question

39. Use Euclid's Division Lemma to show that the square of any positive integer is either of the form 3m or $3m + 1$ for some integer m. [TERM 1, 2012]

Solutions

1. Given, $2^3 \times 3^2$ and $2^2 \times 3^3$

We know, LCM is the product of terms containing highest powers of

$$(2,3) \Rightarrow 2^3 \times 3^3$$

Hence, the correct option is (c). [1]

2. LCM of the given number $= pq$

$$\text{HCF} = \frac{\textit{product of numbers}}{\textit{LCM of numbers}} = \frac{p \times q}{pq} = 1$$

Two integers are co prime when they have no common factor other than 1.

Therefore the H.C.F is 1.

Hence the correct option is (d). [1]

3. The HCF of a and b $= \left(2^2 \times 3^2 \times 5\right)$

$= (4 \times 9 \times 5) = (36 \times 5)$

$= (180)$ [1]

4. HCF (54, b) = 27 and LCM (54, b) = 162

According to the formula,

$\text{LCM}(a, b) \times \text{HCF}(a, b) = a \times b$ [½]

$\Rightarrow 27 \times 162 = 54 \times b$

$\Rightarrow b = \frac{27 \times 162}{54}$

$\Rightarrow b = \frac{162}{2}$

$\Rightarrow b = 81$

So, the other number is 81. [½]

5. The smallest prime number is 2.

And the smallest composite number is 4.

Factors of $2 = 1 \times 2$

$4 = 1 \times 2 \times 2.$ [½]

So the HCF of the smallest prime number and the smallest composite number is 2. [½]

6. $\Rightarrow 2^2 \times 5^3 \times 3^2 \times 17$

$= 2 \times 2 \times 5 \times 5 \times 5 \times 3 \times 3 \times 17$ [½]

$= 10 \times 10 \times 15 \times 51 = 76500$

Hence, the number of zeroes in the end = 2 [½]

7. (b)

7	196
7	28
2	4
2	2
	1

$\Rightarrow 196 = 7^2 \times 2^2$

Sum of exponents = 2 + 2 = 4 [1]

8. (c) $0 \le r < b$ [1]

9. (d)

5	120
3	24
2	8
2	4
2	2
	1

$120 = 2^3 \times 5 \times 3$

Ans. (d) $5 \times 2^3 \times 3$ [1]

10. (a) According to the prime factorisation, 3750 can be written as

$3750 = 5 \times 5 \times 5 \times 5 \times 3 \times 2 = 5^4 \times 3^1 \times 2^1$

It is clear from above, that exponent of 5 in the prime factorisation of 3750 is 4. [1]

11. (d) First subtract the remainders from their respective number,

$1251 - 1 = 1250$

$9377 - 2 = 9375$

$15628 - 3 = 15625$

According to the prime factorisation,

$1250 = 2 \times 5 \times 5 \times 5 \times 5$

$9375 = 3 \times 5 \times 5 \times 5 \times 5 \times 5$

$15625 = 5 \times 5 \times 5 \times 5 \times 5 \times 5$

HCF(1250, 9375, 15625)

$= 5 \times 5 \times 5 \times 5$

$= 625$ [1]

12. (a) As a and b are co-prime then a^3 and b^3 are also co-prime.

We can understand above situation with the help of an example.

Let $a = 3$ and $b = 4$

$a^3 = 3^3 = 27$ and $b^3 = 4^3 = 64$

Clearly, HCF(a, b) = HCF(3, 4) = 1

Then, HCF (a^3, b^3) = HCF(27, 64) = 1 [1]

13. (d) Let us take an example of different powers of 5.

As, $5^1 = 5$; $5^2 = 25$; $5^3 = 125$; $5^4 = 625$

It is clear from above example that 5^n will always end with 5.

Similarly, 6^n will always end with 6.

So, $5^n + 6^n$ will always end with 6.

Also, $2(5^n + 6^n)$ always ends with $2 \times 11 = 22$

i.e., it will always end with 2. [1]

14. (c) According to the property, HCF of two numbers is also a factor of LCM of same two numbers.

Out of all the options, only (c) 500 is not a factor of 2400.

Therefore, 500 cannot be the HCF. [1]

15. (a) Prime factorisation of $92 = 2 \times 2 \times 23$

Prime factorisation of $152 = 2 \times 2 \times 2 \times 19$ [1]

To find HCF, we multiply all the prime factors common to both number:

Therefore, HCF = $2 \times 2 = 4$

16. (c) Let the two consecutive even numbers be 2n and (2n + 2).

Prime factorisation of $2n = 2 \times n$

Prime factorisation of $(2n + 2) = 2 \times (n+1)$

To find HCF, we multiply all the prime factors common to both numbers

Therefore, HCL = 2 [1]

17. (a) We know that HCF × LCM = Product of two numbers

$\Rightarrow$ HCF × LCM = 20 × 50

$\therefore$ HCF × LCM = 1000 [1]

18. (d) Since 6^n is expressed as $(2 \times 3)^n$, it can never end with digit 0 as it does not have 5 in its prime factorisation. [1]

19. (c)

$$p^2 = \frac{32}{50}$$

$$p^2 = \frac{16}{25}$$

$$p = \frac{4}{5}$$

Here, rational number is a number in the form of $\frac{p}{q}$ where p and q are integ ers having no common factor other than 1 and q doesn't equals to 0. [1]

20. (a) LCM of 30 and 70 is 210 and HCF of 30 and 70 is 10.

Hence, (HCF × LCM) of 30 and 70 = 2100 [1]

21. The prime factorization should have 2 and 5 as a common factor for a number to end with the digit zero. [1]

$8^n = (2 \times 2 \times 2)^n$ does not have 5 in its prime factorization.

Hence, 8^n cannot end with the digit zero for any natural number n. [1]

22. We know, by Euclid's Division Lemma,

$a = bq + r,\ 0 \le r < b$

Applying Euclid's Lemma,

Step 1 : Since 240 > 228, we apply the division lemma to 240 and 228, to get $240 = 228 \times 1 + 12$ [1]

Step 2 : Since the remainder $12 \neq 0$, we apply the division lemma to 228 and 12, to get $228 = 12 \times 19 + 0$

The remainder has now become zero.

Since the divisor at this stage is 12, the HCF is 12. [1]

23. We can write $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 + 5$ as

$\Rightarrow 5(1 \times 2 \times 3 \times 4 \times 6 \times 7 + 1)$

$\Rightarrow 5(1 \times 2 \times 3 \times 4 \times 6 \times 7 + 1) = 5 \times 1009$ [1]

Hence we can say that the given number has at least one factor other than 1 and number itself.

$\Rightarrow (5, 1009, 1, 5045)$

Therefore $1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 + 5$ is a composite number. [1]

24. Finding the LCM of 36 and 54,

$36 = 2 \times 2 \times 3 \times 3$

$54 = 2 \times 3 \times 3 \times 3$

LCM $= 2 \times 2 \times 3 \times 3 \times 3 = 108$ [1]

Now it is given that the number is diminished by 5.

This means the least positive will be:

$5 + (\text{LCM of 36 and 54})$

$= 5 + 108$

$= 113$ [1]

Hence, 113 is the least positive integer which on diminishing by 5 is exactly divisible by 36 and 54.

25. 5050 can be factored as,

$5050 = 2 \times 5 \times 5 \times 101$

We can write it as $2 \times 5^2 \times 101$

Here all the factors are prime numbers and can be expressed as product of its prime numbers.

So, Yes it is unique. [2]

26. By using Euclid's division leema

a = bq + r

where, a > b

So, a = 867 and b = 255

867 = 255 × 3 + 102

here, r ≠ 0, Hence, a = 255 and b = 102 [1]

Now, 255 = 102 × 2 + 51

Here, r ≠ 0, Hence, a = 102 and b = 51

102 = 51 × 2 + 0

Here, r = 0

So, HCF of (867, 251) = 51 [1]

27. 6^n

$\Rightarrow (2 \times 3)^n$ [1]

It can be observed that 5 is not in the prime factorisation of 6. Hence for any value of n, 6^n will not be divisible by 5.

$\therefore 6^n$ cannot end with 0 for any natural no. n. [1]

OR

LCM (150, 200)

5	150
5	30
3	6
2	2
	1

5	200
5	40
2	8
2	4
2	2
	1

[1]

$$150 = 2^1 \times 3^1 \times 5^2$$
$$200 = 2^3 \times 5^2$$
$$\text{LCM}(150, 200) = 2^3 \times 3^1 \times 5^2$$
$$= 8 \times 3 \times 25 = 600$$ [1]

28. If any digit has the last digit 10 that means it is divisible by 10.

The factor of 10 = 2 × 5,

So value of 6^n should be divisible by 2 and 5.

Both 6^n is divisible by 2 but not divisible by 5.

So, it can not end with 0. [2]

OR

The prime factorisation of 72 and 120, respectively, is given by:

$72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$

$120 = 2 \times 2 \times 2 \times 3 \times 5 = 2^3 \times 3^1 \times 5^1$

LCM (72, 120) = 360

The common prime factors of 72 and 120 are 2, 2, 2 and 3.

Hence, the HCF of 72 and 120

$= 2 \times 2 \times 2 \times 3 = 24.$

HCF (72, 120) = 24 [2]

29. Let c be any positive number and $d = 3$

Then $c = 3q + r$ for $q \geq 0$

Also, $r = 0, 1, 2$ as $0 \leq r \leq 3$ [1]

Thus, $c = 3q$ or $c = 3q + 1$ or $c = 3q + 2$

$\Rightarrow c^2 = (3q)^2$ or $(3q+1)^2$ or $(3q+2)^2$

$\Rightarrow c^2 = 3 \times (3q^2)$ or $9q^2 + 6q + 1$ or $9q^2 + 12q + 4$

$\Rightarrow c^2 = 3 \times (3q^2)$ or $3(3q^2 + 2q) + 1$ or $3(3q^2 + 4q + 1) + 1$

[1]

$\Rightarrow c^2 = 3m_1$ or $3m_2 + 1$ or $3m_3 + 1$ where

$m_1 = 3q^2, m_2 = 3q^2 + 2q$ and $m_3 = 3q^2 + 4q + 1$

Hence, square of any positive integer is either of $3m$ or $(3m + 1)$ for some integer m. [1]

30. Find the factors of 336 and 54.

$336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7$

$54 = 2 \times 3 \times 3 \times 3$ [1]

HCF of 336 and $54 = 2 \times 3 = 6$

LCM of

336 and $54 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 7 = 3024$ [1]

Product of two numbers $= 336 \times 54 = 18144$

Hence verified. [1]

31. $a = 2160, b = 847$

By Euclid's lemma, given positive integers a and b, there exist unique integers q and r satisfying

$a = bq + r, \ 0 \leq r < b.$ [1]

As $2160 > 847$, we apply the division lemma to 2160 and 847, to get $2160 = 847 \times 2 + 466$

Since the remainder $466 \neq 0$, we apply the division lemma to 847 and 466, and continue the same process till we get remainder 0. [1]

$847 = 466 \times 1 + 381$

$466 = 381 \times 1 + 85$

$381 = 85 \times 4 + 41$

$85 = 41 \times 2 + 3$

$41 = 3 \times 13 + 2$

$3 = 2 \times 1 + 1$

$2 = 1 \times 1 + 1$

$1 = 1 + 0$

As 1 is the HCF of 847 and 2160. 847 and 2160 are the co-primes. [1]

32. Consider 252 and 324. Let, $a = 324$ and $b = 252$ by Euclid's division lemma-

$a = bq + r, \ 0 <$ or $= r < b$

$324 = 252 \times 1 + 72$

$252 = 72 \times 3 + 36$

$72 = 36 \times 2 + 0$ [1]

Therefore, HCF $(252, 324) = 36$

Now consider 36 and 180,here $a = 180$ and $b = 36$. By Euclid's division

$a = bq + r, \ 0 <$ or $= r < b$ [1]

$180 = 36 \times 5 + 0$

Therefore, $HCF(180, 36) = 36$ [1]

33. Pens are sold in pack of 8 and notepads are sold in pack of 12,

LCM of 8 and 12 is:

$8 = 2^3$ and $12 = 2^2 \times 3$ [1]

LCM $= 2^3 \times 3 = 8 \times 3 = 24$

Least number of pack of pen $= \dfrac{24}{8} = 3$ [1]

Least number of pack of notepads $= \dfrac{24}{12} = 2$

Hence, 3 packs of pen and 2 packs of notepads one should buy to get 24 pens and notepads. [1]

34. $3 \times 5 \times 13 \times 46 + 23$

It can be re-written as:

$$3 \times 5 \times 13 \times 2 \times 23 + 23 = 23(3 \times 5 \times 13 \times 2 + 1)$$
$$= 23 \times 391 = 8993$$

[1]

Here 8993 is written as the product of two different numbers 23×391. [1]

It means it has 23 and 391 as its factors other than 1 and 8993.

Hence, it is a composite number. [1]

35. Greatest number of 6 digits is 999999

The numbers given are 18, 24 and 36.

Here LCM of 18, 24, 36.

$18 = 2 \times 3 \times 3 = 2 \times 3^2$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$ [1]

$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$

Now,

The LCM of 18, 24 and 36 $= 2^3 \times 3^2 = 72$

Now dividing 999999 by 72 [1]

$\frac{999999}{72} = 13888$ with remainder 63

And,

$999999 - 63 = 999936$

Thus 999936 is the greatest number 6-digit number divisible by18, 24 and 36. [1]

36. $x^3 + 13x^2 + x - 2$ can be divided by $2x + 1$ as

$$\begin{array}{r|l} & \frac{1}{2}x^2 + \frac{25}{4}x - \frac{21}{8} \\ \hline 2x+1 & x^3 + 13x^2 + x - 2 \\ & x^3 + \frac{1}{2}x^2 \\ & - \quad - \\ \hline & \frac{25}{2}x^2 + x - 2 \\ & \frac{25}{2}x^2 + \frac{25}{4}x \\ & - \quad - \\ \hline & -\frac{21}{4}x - 2 \\ & -\frac{21}{4}x - \frac{21}{8} \\ & + \quad + \\ \hline & \frac{5}{8} \end{array}$$

[1]

Here quotient is $\frac{1}{2}x^2 + \frac{25}{4}x - \frac{21}{8}$ and remainder is $\frac{5}{8}$. [1]

37. The prime factors of:

$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$

$404 = 2 \times 2 \times 101$ [1]

Therefore the HCF = Product of smallest power of each common prime factor = 2 × 2 = 4 [1]

And LCM = Product of greatest power of each prime factor = 2 × 2 × 2 × 2 × 2 × 3 × 101

= 9696

To prove:

HCF × LCM = 101 × 96

Here, HCF × LCM = 4 × 9696

= 38784

101 × 96 = 38784

Hence proved, HCF × LCM = Product of the two given numbers. [1]

38. **HCF by prime Factorization method**

First, we have to find the highest common Factor of 26, 65 and 117

Now let us write the prime factors of 26, 65, and 117.

26 = 2 ×13

65 = 5 ×13

117 = 3 × 3 × 13 [1½]

The common factor of 26, 65, and 117 is 13

Therefore, HCF(26, 65, 117) = 13

LCM by prime factorization method

To calculate the LCM of 26, 65 and 117

First, list the common factors of each number

26 = 2 × 13

65 = 5 × 13

117 = 3 × 3 × 13

LCM = 2 × 5 × 3 × 3 × 13

= 1170 [1½]

39. We know, By Euclid's Division Lemma,

If a and b are two positive integers, then

$a = bq + r$ where $0 \le r < b$ ...(1) [1]

Let a be any positive integer and b = 3, using equation 1, we get,

$a = 3q + r$ where $0 \le r < 3$

We know can be either 0, 1 or 2 [1]

If $r = 0$	If $r = 1$	If $r = 2$
The equation becomes, $a = 3q + 0$ $\Rightarrow a = 3q$ Squaring both sides, $a^2 = (3q)^2$ $\Rightarrow a^2 = 9q^2$ $\Rightarrow a^2 = 3(3q^2)$ Let $m = 3q^2$ $\Rightarrow a^2 = 3m$	The equation becomes, $a = 3q + 1$ Squaring both sides, $a^2 = (3q+1)^2$ $\Rightarrow a^2 = 9q^2 + 6q + 1$ $\Rightarrow a^2 = 3(3q^2 + 2q) + 1$ Let $m = 3q^2 + 2q$ $\Rightarrow a^2 = 3m + 1$	The equation becomes, $a = 3q + 2$ Squaring both sides, $a^2 = (3q+2)^2$ $\Rightarrow a^2 = 9q^2 + 12q + 4$ $\Rightarrow a^2 = 9q^2 + 12q + 3 + 1$ $\Rightarrow a^2 = 3(3q^2 + 4q + 1) + 1$ Let $m = 3q^2 + 4q + 1$ $\Rightarrow a^2 = 3m + 1$

[2]

Hence, square of any positive number can be expressed of the form $3m$ or $3m+1$ for some integer m.

Hence proved.

MULTIPLE CHOICE QUESTIONS

1. Euclid's division lemma states that for two positive integers a and b, there exist unique integers q and r such that a = bq + r, where r must satisfy
   (a) $1 < r < b$ (b) $0 < r \le b$
   (c) $0 \le r < b$ (d) $0 < r < b$
2. The LCM of two numbers is 1200. Which of the following cannot be their HCF?
   (a) 600 (b) 500
   (c) 400 (d) 200
3. If two positive integers a and b are expressible in the form $a = pq^2$ and $b = p^3q$; p, q being prime numbers, then LCM (a, b) is
   (a) pq (b) p^3q^3
   (c) p^3q^2 (d) p^2q^2
4. In question 3, HCF (a, b) is.
   (a) pq (b) p^3q^3
   (c) p^3q^2 (d) p^2q^2
5. If the LCM of a and 18 is 36 and the HCF of a and 18 is 2, then a =
   (a) 2 (b) 3
   (c) 4 (d) 1
6. If HCF of 26 and 169 is 13, then LCM of 26 and 169 be.
   (a) 26 (b) 52
   (c) 338 (d) 13
7. The LCM and HCF of two rational numbers are equal, then the numbers must be
   (a) prime (b) Co-prime
   (c) composite (d) equal
8. If the sum of LCM and HCF of two numbers is 1260 and their LCM is 900 more than their HCF, then the product of two numbers is
   (a) 203400 (b) 194400
   (c) 198400 (d) 205400
9. The least number that is divisible by all the numbers from 1 to 10 (both inclusive) is.
   (a) 10 (b) 100
   (c) 504 (d) 2520
10. The largest number which divides 70 and 125, leaving remainders 5 and 8 respectively is
    (a) 13 (b) 65
    (c) 875 (d) 1750
11. For some integer m, every even integer is of the form
    (a) m (b) m + 1
    (c) 2m (d) 2m + 1
12. For some integer q, every odd integer is of the form
    (a) q
    (b) q + 1
    (c) 2q
    (d) 2q + 1

Answer Keys

1. (c) 2. (b) 3. (c) 4. (a) 5. (c)
6. (c) 7. (d) 8. (b) 9. (d) 10. (a)
11. (c) 12. (d)

Solutions

1. Euclid's division lemma states that for two positive integers a and b there exist unique integers q and r such that a = bq + r, where r must satisfy $0 \le r < b$ **[1]**
2. 500 cannot be their HCF because 1200 can not be completed divided by 500. **[1]**
3. $a = pq^2$
   $b = p^3q$
   LCM (a, b) = p^3q^2. **[1]**
4. $a = pq^2 = p \times q \times q$
   $b = p^3q = p \times p \times p \times q$
   $\therefore$ Required HCF (a, b) = pq **[1]**
5. $\because$ We know that
   LCM × HCF = first number × second number
   $36 \times 2 = a \times 18$
   $\Rightarrow a = \frac{36 \times 2}{18} = 4$ **[1]**
6. $\because$ We know that
   LCM × HCF = First number × second number
   13 × LCM = 26 × 169
   $\text{LCM} = \frac{26 \times 169}{13} = 338$ **[1]**
7. If LCM and HCF of two rational numbers are equal then the numbers must be equal. **[1]**
8. According to the question.
   $\because$ LCM = 900 + HCF
   LCM – HCF = 900...(i)
   and LCM + HCF = 1260 ...(ii)
   Solving eq (i) and (ii), we get,
   2LCM = 2160
   $\Rightarrow \text{LCM} = \frac{2160}{2} = 1080$
   Putting the value of LCM in eq. (ii), we get
   HCF = 1260 – 1080 = 180
   $\because$ Product of two numbers = LCM × HCF
   = 1080 × 180 = 194400 **[1]**
9. Required number = LCM of 1, 2, 3, 4, 5,----10
   $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 \times 5 \times 7 = 2520$ **[1]**
10. Required number
    = HCF of (70 – 5) and (125 – 8)
    = HCF of 65 and 117 = 13 **[1]**
11. Some integer m, every even integer is of the form of 2m. **[1]**
12. for some integer q, every odd integer is of the form (2q + 1). **[1]**

[TOPIC 2] Irrational Numbers, Terminating and Non-Terminating Recurring Decimals

Summary

Irrational Numbers

All real numbers which are not rational are called irrational numbers. $\sqrt{2}$, $\sqrt[3]{3}$, $-\sqrt{5}$ are some examples of irrational numbers.

There are decimals which are non-terminating and non-recurring decimal.

Example: 0.303003000300003…

Hence, we can conclude that

An irrational number is a non-terminating and non-recurring decimal and cannot be put in the form $\frac{p}{q}$ where p and q are both co-prime integers and $q \neq 0$.

Decimal Representation of Rational Numbers

Theorem: Let $x = \frac{p}{q}$ be a rational number such that $q \neq 0$ and prime factorization of q is of the form $2^n \times 5^m$ where m, n are non-negative integers then x has a decimal representation which terminates.

For example : $0.275 = \frac{275}{10^3} = \frac{5^2 \times 11}{2^3 \times 5^3} = \frac{11}{2^3 \times 5} = \frac{11}{40}$

Theorem: Let $x = \frac{p}{q}$ be a rational number such that $q \neq 0$ and prime factorization of q is not of the form $2^m \times 5^n$, where m, n are non-negative integers, then x has a decimal expansion which is non-terminating repeating.

For example : $\frac{5}{3} = 1.66666...$

Rational number	Form of prime factorisation of the denominator	Decimal expansion of rational number
$x = \frac{p}{q}$, where p and q are coprime and $q \neq 0$	$q = 2^m 5^n$ where n and m are non-negative integers	terminating
	$q \neq 2^m 5^n$ where n and m are non-negative integers	non-terminating

- *If the denominator is of the form $2^m \times 5^n$ for some non negative integer m and n, then rational number has terminating decimal otherwise non terminating.*

PREVIOUS YEARS' EXAMINATION QUESTIONS

TOPIC 2

1 Mark Questions

1. The prime factorization of the denominator of the rational number expressed as $46.\overline{123}$ is:
   (*a*) $2^m \times 5^n$ Where m and n are integers
   (*b*) $2^m \times 5^n$ Where m and n are positive integers
   (*c*) $2^m \times 5^n$ Where m and n are rational numbers
   (*d*) Not of the form $2^m \times 5^n$ where m and n are non-negative integers. **[TERM 1, 2011]**

2. The decimal expansion of $\frac{6}{1250}$ will terminate after how many places of decimal?
   (*a*) 1 (*b*) 2
   (*c*) 3 (*d*) 4 **[TERM 1, 2011]**

3. Decimal expansion of $\frac{23}{(2^3 5^2)}$ will be:
   (*a*) Terminating
   (*b*) Non- terminating
   (*c*) Non terminating and repeating.
   (*d*) Non-terminating and non-repeating
   [TERM 1, 2012]

4. Find a rational number between $\sqrt{2}$ and $\sqrt{7}$.
   [2019]

5. $\frac{57}{300}$ is a
   (a) non-terminating and non-repeating decimal expansion
   (b) terminating decimal expansion after 2 places of decimals
   (c) terminating decimal expansion after 3 places of decimals
   (d) non-terminating but repeated decimal expansion **[Basic Term 1, 2022]**

6. $5.\overline{213}$ can also be written as
   (a) 5.213213213...
   (b) 5.2131313...
   (c) 5.213
   (d) 5213/1000 **[Basic Term 1, 2022]**

7. The decimal expansion of $\frac{13}{2 \times 5^2 \times 7}$ is
   (a) terminating after 1 decimal place
   (b) non-terminating and non-repeating
   (c) terminating after 2 decimal places
   (d) non-terminating but repeating
   [Basic Term 1, 2022]

8. **Assertion – Reason Based Questions:** A statement of Assertion (A) is followed by a statement of Reason (R)

   Statement A (Assertion): If $5 + \sqrt{7}$ is a root of a quadratic equation with rational co-efficients, then its other root is $5 - \sqrt{7}$.

   Statement R (Reason): Surd roots of a quadratic equation with rational co-efficients occur in conjugate pairs.

   Choose the correct option out of the following:
   (a) Both Assertion (A) and Reason (R) are true; and Reason (R) is the correct explanation of Assertion (A).
   (b) Both Assertion (A) and Reason (R) are true; but Reason (R) is not the correct explanation of Assertion (A).
   (c) Assertion (A) is true but Reason (R) is false.
   (d) Assertion (A) is false but Reason (R) is true.
   [Standard, 2023]

9. The number $\left(5-3\sqrt{5}+\sqrt{5}\right)$ is :

(a) an integer

(b) a rational number

(c) an irrational number

(d) a whole number [Basic, 2023]

2 Marks Questions

10. What can you say about the prime factorization of the denominator of the rational number 0.134 when written in the form $\frac{p}{q}$. Is it of form $2^m \times 5^n$? If yes, write the values of m and n. [TERM 1, 2013]

11. Find the smallest positive rational number by which $\frac{1}{7}$ should be multiplied so that its decimal expansion terminates after 2 places of decimal. [TERM 1, 2011]

12. Show that $\left(\sqrt{3}+\sqrt{5}\right)^2$ is an irrational number. [TERM 1, 2015]

13. Write down the decimal expansion of $\frac{76}{6250}$, without actual division. [TERM 1, 2016]

14. Find how many integers between 200 and 500 are divisible by 8. [TERM 1, 2017]

15. Given that $\sqrt{2}$ is irrational, prove that $\left(5+3\sqrt{2}\right)$ is an irrational number. [TERM 1, 2017]

16. Prove that $\sqrt{3}+\sqrt{2}$ is irrational. [TERM 1, 2011]

3 Marks Question

17. Prove that (3 + 2 $\sqrt{5}$) is an irrational number, given that $\sqrt{5}$ is an irrational number. [2019]

18. Prove that $\sqrt{3}$ is an irrational number. [Basic, 2020]

19. Prove that $\sqrt{2}$ is an irrational number. [Standard, 2023]

4 Marks Questions

20. Define irrational number and prove that $3+\sqrt[2]{5}$ is an irrational number. [TERM 1, 2017]

21. Prove that $\sqrt{5}$ is an irrational number. [Standard, 2020]

Solutions

1. As the decimal expansion $46.\overline{123}$ is a non-terminating repeating, the given number is a rational number of the form $\frac{p}{q}$ where q is not of the form $2^m \times 5^n$.

Let $x = 46.\overline{123}$ ...(1)

$1000x = 46123.\overline{123}$ ...(2)

$(2)-(1) \Rightarrow \frac{46077}{999} = x$

Hence, the correct option is (d). [1]

2. Express 6 and 1250 as a product of prime factors.

$$\frac{6}{1250} = \frac{2\times 3}{2\times 5^4}$$

$$\Rightarrow \frac{6}{1250} = \frac{2\times 3}{2\times 5^4}\times\frac{2^3}{2^3} = \frac{48}{5^4\times 2^4}$$

$$\Rightarrow \frac{6}{1250} = \frac{48}{(5\times 2)^4} = \frac{48}{10000} = 0.0048$$

Hence, decimal expansion terminates after 4 places of decimal. The correct option is (d). [1]

3. We know by a theorem that,

If $x=\frac{p}{q}$ be a rational number, such that the prime factorization of q is in the form 2^n5^m, where n, m are non-negative integers. Then x has a decimal expansion which terminates. [½]

Hence, Decimal expansion of $\frac{23}{2^3 5^2}$ will be terminating [½]

So, the correct option is (a).

4. $\because \sqrt{2}=1.414$ and $\sqrt{7}=2.645$ [½]

$\therefore$ Rational number between $\sqrt{2}$ and $\sqrt{7}$ = 2 [½]

5. (b) Terminating decimal expansion after 2 places of decimals.

Here $\frac{57}{300}$ can be written as $= \frac{57}{2^2\times 3^1\times 5^2}$

Further, it can be written as

$$\frac{19}{2^2\times 5^2} = \frac{19}{100} = 0.19$$

Since, the denominator is of the form $2^m \times 5^n$, the decimal expansion will be terminating.

Therefore, it is terminating decimal expansion after 2 decimal places. [1]

6. (a) Bar present on 213 in $5.\overline{213}$ means 213 is repeated multiple times. [1]

7. (d) The denominator of $\frac{13}{2\times 5^2 \times 7}$ is not of the form $2^m \times 5^n$, so, its decimal expansion is non-terminating but repeating. [1]

8. (a) [1]

9. (c) an irrational number [1]

10. Let x = 0.134 ....(1)

Now, $100x = 134.134$ ....(2) [1]

Subtract eqn (1) from (2) We get,

$999x = 134$

$$x = \frac{134}{999}$$

$$x = \frac{134}{9(111)}$$

$$x = \frac{134}{3^2(111)}$$

The above expression can-not be written as $2^m \times 5^n$. [1]

11. Decimal expansion of a any rational number terminates if the denominator of the rational number is in the form 2^n5^m

Let the number multiplied by $\frac{1}{7}$ be x,

$$\frac{1}{7}\times x = \frac{1}{2^n5^m}$$

$$\therefore x = \frac{7}{2^n5^m}$$ [1]

Now here when $n = 2$ and $m = 0$

$$x = \frac{7}{2^25^0} = \frac{7}{4}$$

When $n = 0$, $m = 2$

Now if we put $n = 2$ and $m = 2$,

We have $x = \frac{7}{2^25^2} = \frac{7}{100}$

Hence we can see that $\frac{7}{100}$ is smallest possible rational number we multiply by $\frac{1}{7}$ so that the decimal expansion will terminate after 2 decimal places. [1]

12. Let $\left(\sqrt{3}+\sqrt{5}\right)^2$ is a rational number.

$\Rightarrow \left(\sqrt{3}+\sqrt{5}\right)^2 = \frac{p}{q}$ Where p, q are co-prime

Using $(a+b)^2 = a^2+b^2+2ab$ we get,

$$\left(\sqrt{3}\right)^2 + \left(\sqrt{5}\right) + 2\sqrt{3}\sqrt{5} = \frac{p}{q}$$ [1]

$$\Rightarrow 3+5+2\sqrt{15} = \frac{p}{q} \Rightarrow 8+2\sqrt{15} = \frac{p}{q}$$

$$\Rightarrow 2\sqrt{15} = \frac{p}{q} - 8 \Rightarrow \sqrt{15} = \frac{1}{2}\left(\frac{p}{q}-8\right)$$

$$\Rightarrow \sqrt{15} = \left(\frac{p}{2q}-4\right)$$

The RHS is the difference of two rational numbers.

Therefore LHS will also be rational.

But we know that $\sqrt{15}$ is irrational.

So our assumption is wrong. [1]

Hence, $\left(\sqrt{3}+\sqrt{5}\right)^2$ is an irrational number.

13. $\frac{76}{6250} = \frac{76}{5^5\times 2}$

Here,

$\frac{76}{6250}$ is in the form of $\frac{p}{q}$ and q is in the form of 2^n5^m where n and m are non - negative integers. [1]

Hence $\frac{76}{6250}$ has terminating decimal expression.

Now,

$$\frac{76}{6250} = \frac{76}{5^5\times 2} = \frac{76\times 2^4}{5^5\times 2\times 2^4} = \frac{76\times 16}{10^5} = \frac{1216}{100000}$$

$= 0.01216$

Thus the decimal expansion of $\frac{76}{6250}$ is 0.01216.

[1]

14. The first number that is divisible by 8 between 200 and 500 is 208 and the last number that is divisible by 8 are 496.

So, the sequence will be 208, 216, 224 .... 496.

Common difference $d = 8$

First term $a = 208$ [1]

Let there be n terms is the sequence

Using the formula $a_n = a+(n-1)d$

Where $a_n = 496$, $a = 208$ and $d = 8$

$496 = 208 + (n-1)(8)$

$(n-1)8 = 288$

$n - 1 = 36$
$n = 37$
Hence, between 200 and 500 there are 37 integers that are divisible by 8. [1]

15. Suppose $\left(5+3\sqrt{2}\right) = \frac{p}{q}$

Now assume $\left(5+3\sqrt{2}\right)$ is a rational number.

Therefore p and q should be co-prime numbers.

[1]

$\left(5+3\sqrt{2}\right) = \frac{p}{q}$

$\Rightarrow \frac{p}{q} - 5 = 3\sqrt{2}$

$\Rightarrow \frac{p}{3q} - \frac{5}{3} = \sqrt{2}$

$\Rightarrow \frac{p-5}{3q} = \sqrt{2}$

Since $\sqrt{2}$ is irrational number.

Thus the assumption is incorrect and hence $\left(5+3\sqrt{2}\right)$ is an irrational number.

Hence proved. **[1]**

16. Let $\sqrt{3}$ is a rational number. So, two integers a and b can be found so that $\sqrt{3} = \frac{a}{b}$

Assume that a and are co-prime.

$\Rightarrow a = \sqrt{3}b$

Squaring both the sides,

$\Rightarrow a^2 = 3b^2$ **[1]**

So, a^2 is divisible by 3 and it can be said that a is divisible by 3.

Let $a^2 = 3c$, where c is an integer.

$a^2 = 3b^2$

$\Rightarrow (3c)^2 = 3b^2 \Rightarrow b^2 = 3c^2$

So, b^2 is divisible by 3 and it can be said that b is divisible by 3.

This means that a and b have 3 as a common factor which is a contradiction to fact that a and b are co-prime.

Hence, $\sqrt{3}$ cannot be expressed as $\frac{p}{q}$ or $\sqrt{3}$ is irrational.

Similarly, $\sqrt{2}$ is irrational. The sum of two irrational numbers is an irrational number.

$\sqrt{3}+\sqrt{2}$ is sum of two irrational numbers, hence it is an irrational number.

Hence proved. **[1]**

17. If possible, let a = $(3 + 2\sqrt{5})$ be a rational number
On squaring both sides, we get
$a^2 = (3 + 2\sqrt{5})^2$
$\Rightarrow a^2 = 9 + 20 + 12\sqrt{5}$
$\Rightarrow a^2 = 29 + 12\sqrt{5}$

$\Rightarrow \sqrt{5} = \frac{a^2 - 29}{12}$ ...(i) [1]

since 'a' is a rational number,

$\therefore \frac{a^2-29}{12}$ is also a rational number

$\Rightarrow \sqrt{5}$ is a rational number [1]

but It is given that $\sqrt{5}$ is an irrational number.
Hence, it is a contradiction
So, $3 + 2\sqrt{5}$ is an irrational number. [1]

18. Let $\sqrt{3}$ be a rational number, then its simplest form is $\frac{a}{b}$ (where a and b are *co* - primes)

$\left(\sqrt{3}\right)^2 = \frac{a^2}{b^2}$

$\Rightarrow a^2 = 3b^2$ ------ (1)
$\Rightarrow$ 3 divides a^2
$\Rightarrow$ 3 divides a [1]
Let a = 3c, for some integer c
Putting a = 3c in (1),
$3b^2 = 9c^2$
$\Rightarrow b^2 = 3c^2$
$\Rightarrow$ 3 divides b^2
$\Rightarrow$ 3 divides b [1]
Thus, 3 is a common factor of a and b

But this is not possible as a and b are co-primes

$\Rightarrow$ Our assumption that $\sqrt{3}$ is rational is wrong

$\Rightarrow$ $\sqrt{3}$ is irrational [1]

19. Let assume on the contrary that $\sqrt{2}$ is a rational number.

Then, there exists positive integer a and b such that $\sqrt{2} = \frac{a}{b}$ where, a and b are co primes i.e. their HCF is 1.

$\Rightarrow$ $\sqrt{2} = \left(\frac{a}{b}\right)^2$

$\Rightarrow$ $2 = \frac{a^2}{b^2}$

$\Rightarrow$ $a^2 = 2b^2$

$\Rightarrow$ a^2 is multiple of 2

$\Rightarrow$ a is a multiple of 2 ...(i) [1]

$\Rightarrow$ a = 2c for some integer c.

$\Rightarrow$ $a^2 = 4c^2$

$\Rightarrow$ $2b^2 = 4c^2$

$\Rightarrow$ $b^2 = 2c^2$

$\Rightarrow$ b^2 is a multiple of 2

b is a multiple of 2...(ii) [1]

From (i) and (ii), a and b have at least 2 as a common factor. But this contradicts the fact that a and b are co-prime. This means that $\sqrt{2}$ is an irrational number. [1]

20. **Irrational numbers:** are those numbers that cannot be written in form $\frac{p}{q}$ where p and q are integers and $q \neq 0$. In other words, these are the numbers whose decimal expansion is non-terminating and non-repeating.

Let $3 + \sqrt[2]{5}$ be a rational number **[1]**

$\therefore$ We can find two integers $a, b\ (b \neq 0)$ such that

$3 + \sqrt[2]{5} = \frac{a}{b}$ **[1]**

$\sqrt[2]{5} = \frac{a}{b} - 3$

Since a and b are integers, $\frac{a}{b} - 3$ is also a rational number and hence $\sqrt[2]{5}$ should be rational. **[1]**

This contradicts the fact that $\sqrt[2]{5}$ is irrational.

Therefore, our assumption s wrong and hence, $3 + \sqrt[2]{5}$ is an irrational number. **[1]**

21. Let $\sqrt{5}$ be a rational number.

Then it is of the form $\frac{a}{b}$, where a and b are co-prime.

Now, $\sqrt{5} = \frac{a}{b}$

$\Rightarrow$ $5 = \frac{a^2}{b^2}$

$\Rightarrow$ $a^2 = 5b^2$ ...(i)

$\Rightarrow$ 5 divides a^2

$\Rightarrow$ 5 divides a [1½]

Let $a = 5c$ for some integer c

Putting $a = 5c$ in (i),

$5b^2 = 25\,c^2$

$\Rightarrow$ $b^2 = 5\,c^2$

$\Rightarrow$ 5 divides b^2

$\Rightarrow$ 5 divides b [1½]

Thus 5 is a common factor of a and b.

But this is not possible as a and b are co-primes.

$\Rightarrow$ Our assumption that $\sqrt{5}$ is rational is wrong.

$\Rightarrow$ $\sqrt{5}$ is an irrational number. [1]

MULTIPLE CHOICE QUESTIONS

1. Which of the following number are irrational.
   (*a*) $\sqrt{25}$
   (*b*) $\sqrt{9}$
   (*c*) $\sqrt{5}$
   (*d*) 2
2. The number of decimal places after which the decimal expansion of the rational number $\frac{23}{2^2 \times 5}$ will terminate, is
   (*a*) 1
   (*b*) 2
   (*c*) 3
   (*d*) 4

3. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after
   (*a*) one decimal place
   (*b*) two decimal place
   (*c*) three decimal place
   (*d*) four decimal place
4. Which of the following rational numbers have terminating decimal?
   (*a*) $\frac{5}{18}$ (*b*) $\frac{16}{225}$
   (*c*) $\frac{7}{250}$ (*d*) $\frac{2}{21}$
5. $3.\overline{27}$ is
   (*a*) an integer
   (*b*) a natural number
   (*c*) a rational number
   (*d*) an irrational number
6. The smallest number by which $\sqrt{27}$ should be divided so as to get a rational number.
   (*a*) $\sqrt{27}$
   (*b*) $\sqrt{3}$
   (*c*) $3\sqrt{3}$
   (*d*) 3
7. The smallest rational number by which $\frac{1}{3}$ should be multiplied so that its decimal expansion terminates after one place of decimal, is
   (*a*) $\frac{3}{10}$
   (*b*) $\frac{1}{10}$
   (*c*) $\frac{3}{100}$
   (*d*) 3
8. The decimal expansion of the rational number $\frac{33}{15}$ will terminate after.
   (*a*) one decimal place
   (*b*) two decimal place
   (*c*) three decimal place
   (*d*) More then three decimal place
9. $\sqrt{3}+\sqrt{2}$ is
   (*a*) rational number
   (*b*) irrational number
   (*c*) prime number
   (*d*) composite number
10. The decimal expansion of the rational number $\frac{14587}{1250}$ will terminate after.
   (*a*) one decimal place
   (*b*) two decimal place
   (*c*) three decimal place
   (*d*) four decimal place

Answer Keys

1. (*c*)	2. (*b*)	3. (*d*)	4. (*c*)	5. (*c*)
6. (*b*)	7. (*a*)	8. (*a*)	9. (*b*)	10. (*d*)

Solutions

1. $\sqrt{25} = 5$

   $\sqrt{9} = 3$

   $\sqrt{5} = 2.236067$

   So $\sqrt{5}$ is an irrational number. [1]

2. $\frac{23}{2^2 \times 5} = \frac{23}{20} = 1.15$

   So, the given expression will terminate after 2 decimal place. [1]

3. $\frac{14587}{1250} = 11.6696$

   So, the given expression will terminate after 4 decimal place. [1]

4. $\frac{5}{18} = 0.2777...$

   $\frac{16}{225} = 0.071111...$

   $\frac{7}{250} = 0.028$

   $\frac{2}{21} = 0.0952380...$

   So, $\frac{7}{250}$ have terminating decimal. [1]

5. $3.\overline{27}$

Let x = $3.\overline{27}$

Then x = 3.2727 ...(i)

On multiplying by 100 on both side.

100x = 327.27---

$\Rightarrow$ 100x = 324 + 3.272727...

$\Rightarrow$ 100 x = 324 + x (from eq. (i))

$\Rightarrow$ 100x − x = 324

$\Rightarrow$ 99x = 324

$\Rightarrow x = \frac{324}{99} = \frac{36}{11}$

So, $3.\overline{27}$ is a rational number. **[1]**

6. $\because \sqrt{27} = \sqrt{3 \times 3 \times 3}$

$= 3\sqrt{3}$

On divide by $\sqrt{3}$

$\frac{3\sqrt{3}}{\sqrt{3}} = 3$ (rational number)

So, $\sqrt{27}$ is divided by $\sqrt{3}$ to get a rational number. **[1]**

7. $\because \frac{1}{3} = \frac{1}{3} \times \frac{3}{10} = \frac{1}{10} = 0.1$

$\therefore \frac{1}{3}$ should be multiplied by $\frac{3}{10}$, so that its decimal expansion terminates after one place of decimal. **[1]**

8. $\frac{33}{15} = 2.2$

So, the given expression is terminating after one decimal place. **[1]**

9. $\because \sqrt{3}$ is an irrational number and $\sqrt{2}$ is also an irrational number and addition of two irrational number is also irrational $\therefore \left(\sqrt{3} + \sqrt{2}\right)$ is an irrational. **[1]**

10. $\frac{14587}{1250} = 11.6696$

So, the given expression terminates after four decimal place. **[1]**